CE 291 Project Presentation

Optimal Control for Vehicle Maneuvering

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Project Goal

- Model the dynamics of a vehicle with appropriate inputs

- Find the inputs such that the vehicle gets to the destination in minimal time

Roadmap

1) Formulate Dynamics of Vehicle
2) Set Bounds on Inputs
3) Define Roadway Geometry
4) Define Problem as Level Set Problem
5) Compute Optimal Hamiltonian and Control
6) Find Partials
7) Use Level Set Toolbox to Compute Solution
8) Compute Gradient on Level Set Function
9) Compute Optimal Control
10) Find Optimal Path

1. Formulate Dynamics

- Start with dynamic formulation

$$\underline{x} = \begin{bmatrix} x \\ y \\ v_x \\ v_y \\ \theta \end{bmatrix}$$

- Inputs: G,ρ
- Parameter: m

$$\dot{\underline{x}} = \begin{bmatrix} v_x \\ v_y \\ \frac{G}{m}\cos(\theta) \\ \frac{G}{m}\sin(\theta) \\ \rho \end{bmatrix}$$

- Dimension of problem too high to use level set formulation

1. Formulate Dynamics

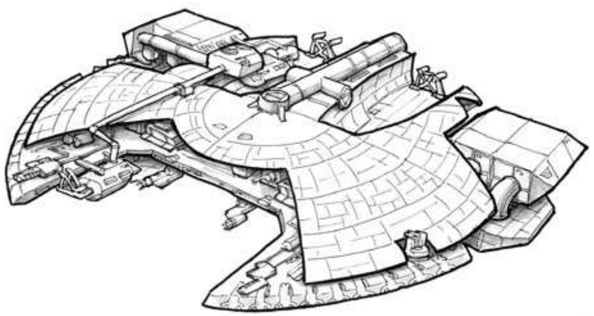
- Reduce to kinematics formulation
- Inputs: v, ρ
- Lower dimension but significant trade offs
- Vehicle can now choose its speed instantaneously

$$\underline{x} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

$$\dot{\underline{x}} = \begin{bmatrix} v\cos(\theta) \\ v\sin(\theta) \\ \rho \end{bmatrix}$$

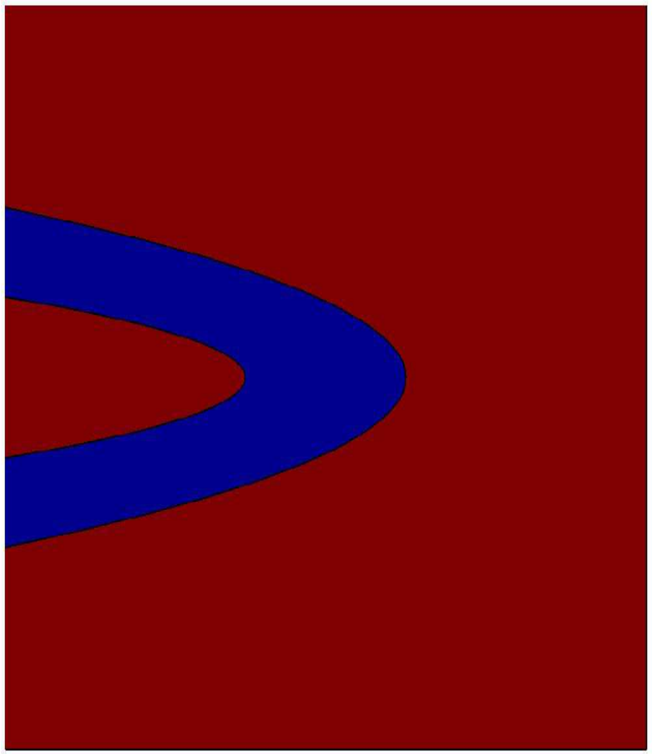
2. Set Bounds on Inputs

- We now have a kinematics model, so any parameters will do
- For this problem, I have chosen: $v \in [0,3]$



$$\rho \in \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$$

3. Define Roadway Geometry



- Started with a simple U-turn roadway modeled by a quadratic
- A function L(x,y) returns 1 inside the roadway and 1e9 outside the roadway

4. Define Level Set Problem

- Suppose we say the car starts at the origin inside some small circle with radius, r_{small}
- The initial set the car can get to at t = 0 can be described by:

$$\{(x, y, \theta), \sqrt{x^2 + y^2} \leq r_{small}\}$$

- However, we can describe the same set implicitly as

$$\{(x, y, \theta), \phi(\underline{x}, t = 0) = \sqrt{x^2 + y^2} - r_{small} \leq 0\}$$

- Where $\phi(\underline{x}, t)$ defines the reachable set for any time, t

4. Define Level Set Problem

- It turns out that $\phi(\underline{x},t)$ is the solution to the Hamilton-Jacobi equation

$$\frac{\partial \phi}{\partial t} + H\left(\underline{x}, \nabla \phi\right) = 0$$

$$\phi\left(\underline{x}, 0\right) = \sqrt{x^2 + y^2} - r_{small} \leq 0$$

- Where H($\underline{x}$,Δϕ) is the Hamiltonian:

$$H\left(\underline{x}, p\right) = \frac{p^T f\left(\underline{x}, u\right)}{L\left(x, y\right)} \quad p = \nabla \Phi \quad u = \begin{bmatrix} v \\ \rho \end{bmatrix} \quad f\left(\underline{x}, u\right) = \underline{\dot{x}}$$

5. Compute H*, v*, and ρ*

- Since the car is trying to maximize its distance from the reachable set, the optimal controls are those that maximize H

$$H^*(\underline{x}, p) = \max_{V \in [v_{\min}, v_{\max}]} \max_{\rho \in [-\rho_{\max}, \rho_{\max}]} \frac{1}{L(x,y)} \left(p_1 v \cos(\theta) + p_2 v \sin(\theta) + p_3 \rho \right)$$

- This gives the following optimal controls:

$$v^* = \frac{v_{\max} + v_{\min}}{2} + \operatorname{sgn}\left(p_1 \cos(\theta) + p_2 \sin(\theta)\right)\left(\frac{v_{\max} - v_{\min}}{2}\right)$$

$$\rho^* = \operatorname{sgn}(p_3)\rho_{\max}$$

- We can plug our optimal controls into H to get H*

6. Compute partials

- Straightforward to compute partials with respect to costate

- $$\frac{\partial H^*}{\partial p_1} = \frac{1}{L(\underline{x})}\begin{pmatrix} \frac{1}{2}(v_{\max} - v_{\min})\cos(\theta) \\ -\frac{1}{2}(v_{\max} + v_{\min})\cos(\theta) \end{pmatrix}$$

- $$\frac{\partial H^*}{\partial p_2} = \frac{1}{L(\underline{x})}\begin{pmatrix} \frac{1}{2}(v_{\max} - v_{\min})\sin(\theta) \\ -\frac{1}{2}(v_{\max} + v_{\min})\sin(\theta) \end{pmatrix}$$

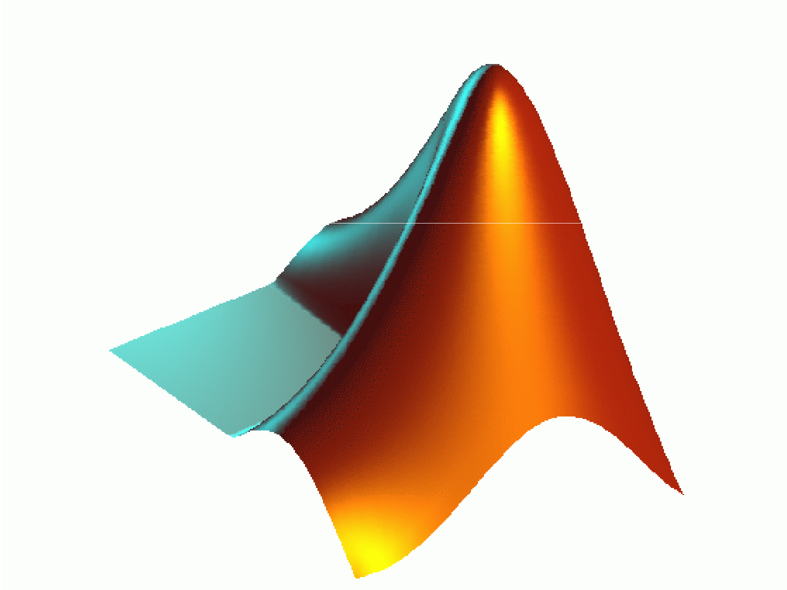
- $$\frac{\partial H^*}{\partial p_3} = \frac{1}{L(\underline{x})}(\rho_{\max})$$

7. Level Set Toolbox

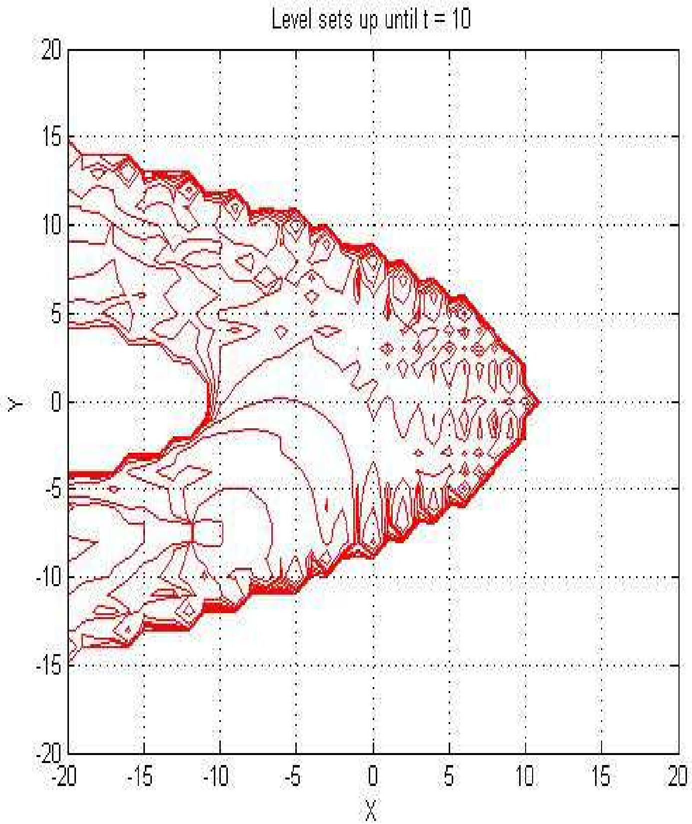
- With the expressions that we derived we can plug into the Level Set Toolbox for MATLAB written by Ian Mitchell et. al

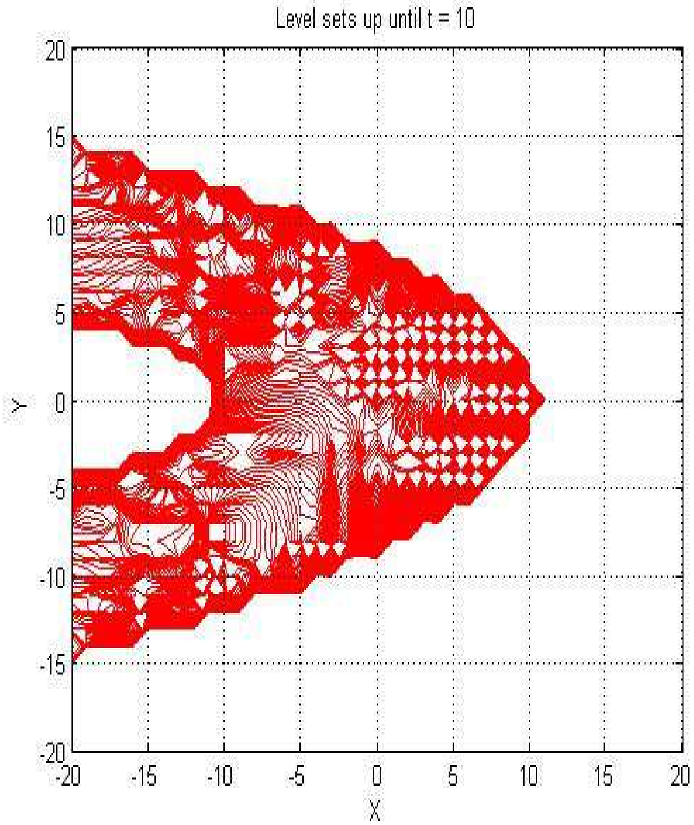
- Initial results were less than ideal

- Numerical anomalies created erratic and unusable level set boundaries



7. Level Set Toolbox (results)



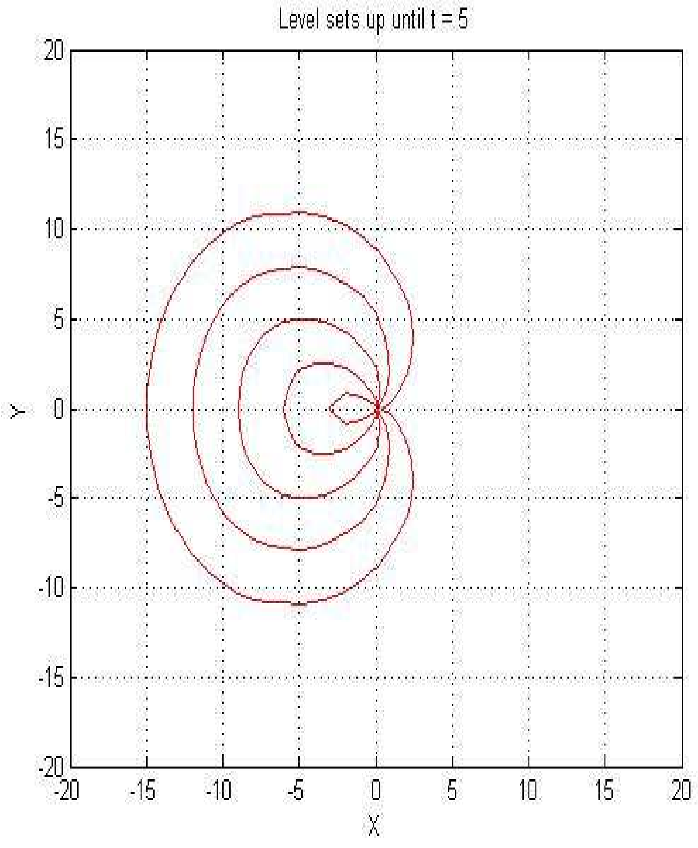


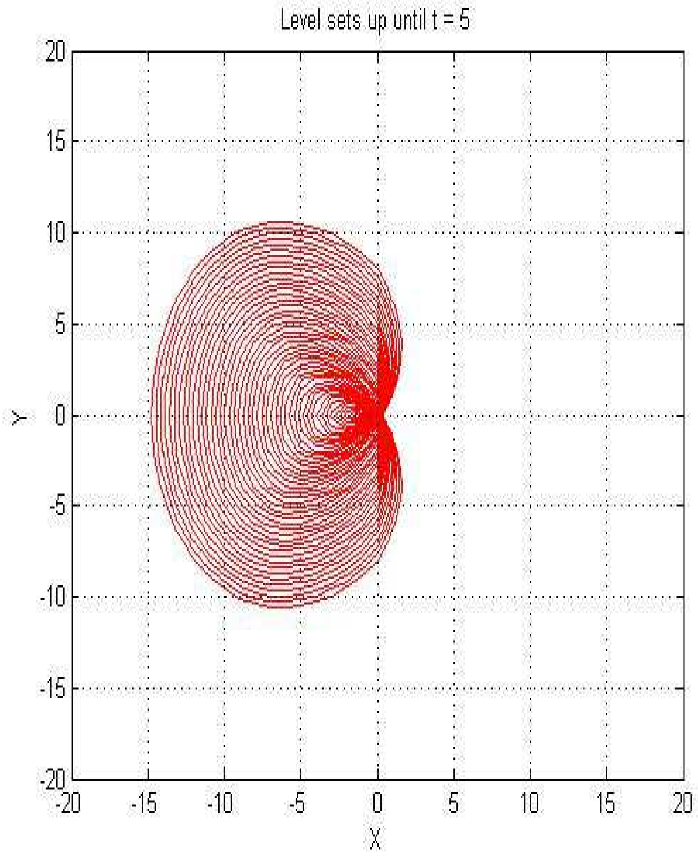
7. Level Set Toolbox

- Numerical anomalies persisted despite changing the increment in both time and space, as well as the weight value assigned to locations outside the roadway, and also level of accuracy of integration
- As a result, I was forced scrap the roadway and try the method in an open field (i.e. L(x,y) = 1 everywhere)

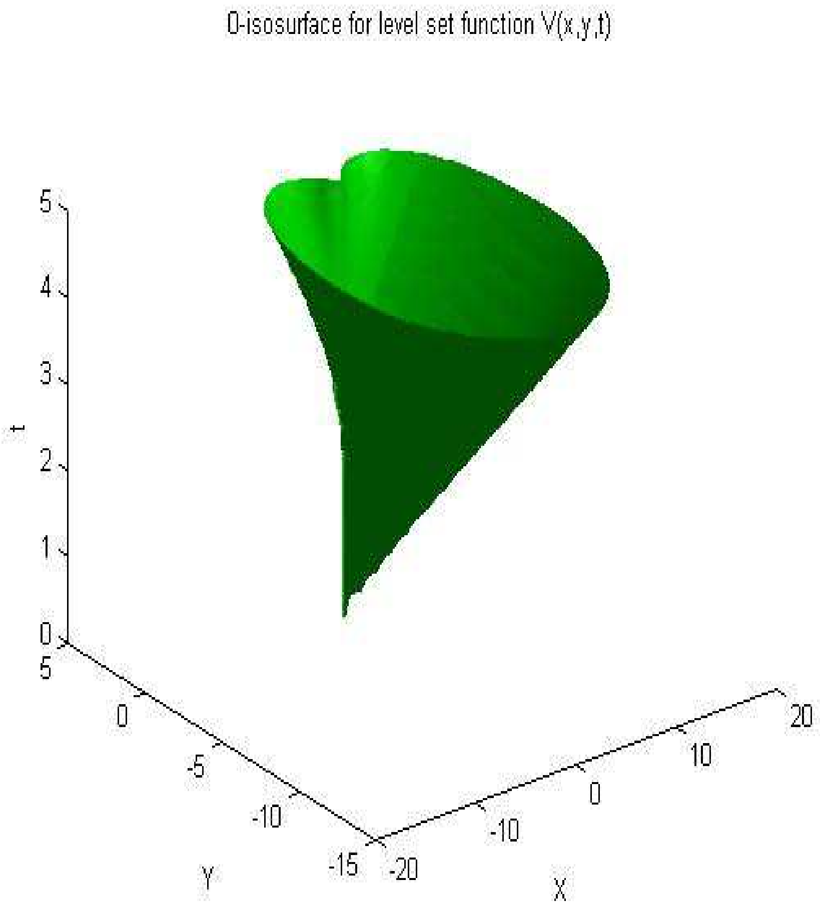
- Possible Reasons:
  - Incorrect formulation
  - Mistakes in derivation or coding
  - Numerical problems innate in Level Set method
  - Fixing problems would require unreasonable changes in level of refinement (i.e. advance time by 1e-9 or discretize space in 1e-9 or both)

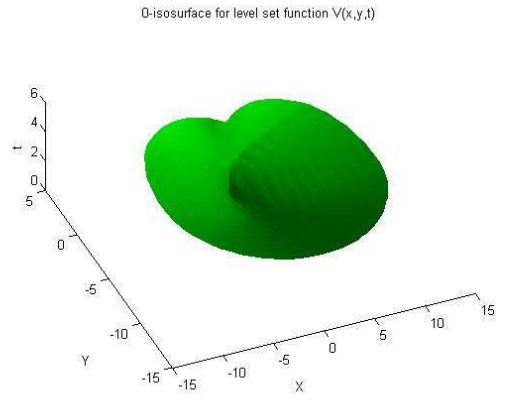
7. Level Set Toolbox (results)

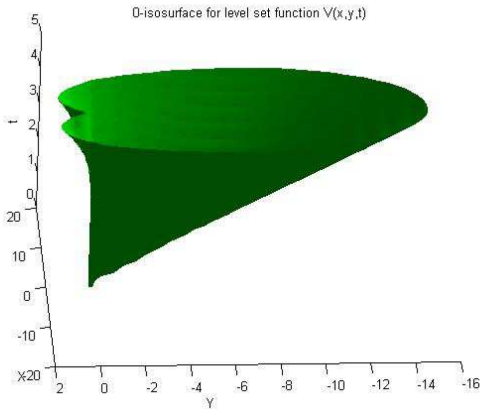




7. Level Set Toolbox (results)

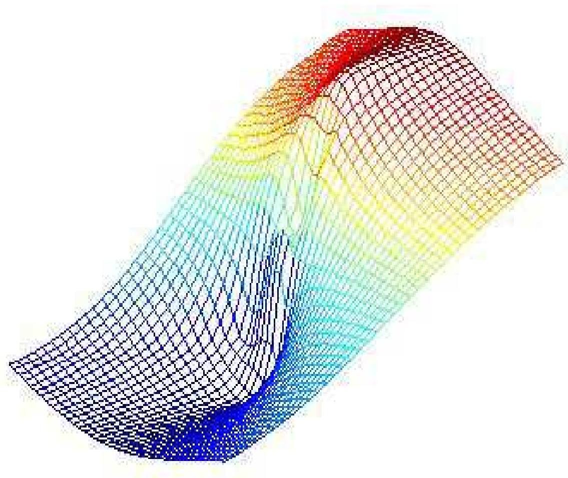






8. Compute Gradient

- For every time, t, computed the gradient of level set function, ϕ
- Used MATLAB function, gradient, which numerically computes single point difference
- Computation ‘massaged’ to output a 4 dimensional grid in which each element is the gradient vector
- To the right is a picture of a 3 dimensional slice of the x component of the gradient.



9. Compute Optimal Control

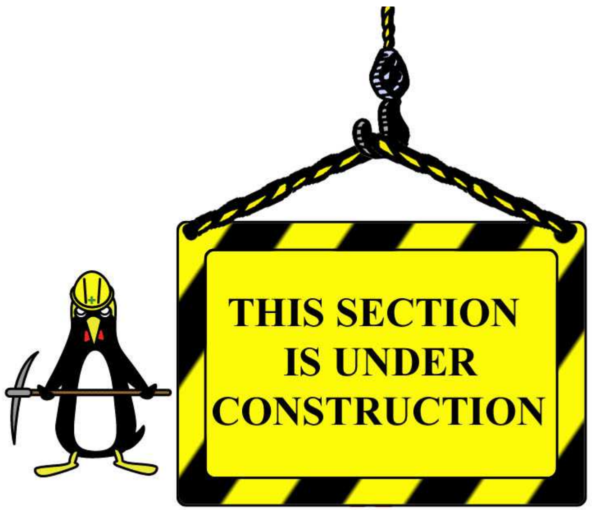
- Recall p_i is the i^{th} component of the gradient
- With the gradient generated for all states (x, y, θ) and for all time, t, we can compute the optimal control for all states and times by plugging in the value of the gradient into the expressions of v* and ρ*
- Notice that for v* and ρ* there is always a 'max-out strategy' (i.e. you are either stopped or going full speed, turning as fast as possible to the right or to the left or going straight)

$$v^* = \frac{v_{\max} + v_{\min}}{2} + \operatorname{sgn}\left(p_1 \cos(\theta) + p_2 \sin(\theta)\right)\left(\frac{v_{\max} - v_{\min}}{2}\right)$$

$$\rho^* = \operatorname{sgn}(p_3)\rho_{\max}$$

10. Find Optimal Path

- Algorithm shown at right
- Unsure about correctness
- Numerical difficulties and extensive ‘massaging’ expected (current state of project)



1) Start at x0, y0, theta0, at time t = 0;
2) Compute optimal control for that point;
3) Use Euler or RK4 and dt to advance state;
4) Assign next value of x, y, and theta and advance t by dt;
5) Return to 2) until converged to desired point; if converged go to 6);
6) History of (x,y) points is best path, t is best time possible

References

- “A Time-Dependent Hamilton-Jacobi Formulation of Reachable Sets for Continuous Dynamic Games”. Ian Mitchell, Alexandre Bayen, and Claire Tomlin.
- “A differential game formulation of alert levels in ETMS data for high altitude traffic”. Alexandre Bayen, Shriram Santhanam, Ian Mitchell, and Claire Tomlin.
- “A Toolbox of Level Set Methods”. Ian Mitchell.
- The Toolbox can be downloaded at: http://www.cs.ubc.ca/~mitchell/ToolboxLS/

The End

Questions?