A general phase transition model for vehicular traffic

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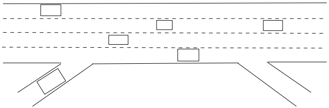
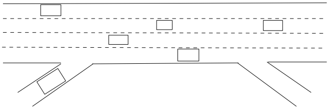
May 4th 2009

Macroscopic models of traffic



- How to model traffic flow on a stretch of highway ?

Macroscopic models of traffic



- How to model traffic flow on a stretch of highway ?
- Assumptions:

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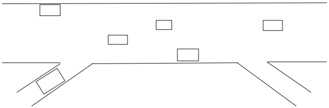

- How to model traffic flow on a stretch of highway ?
- Assumptions:
  - One lane

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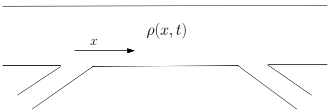
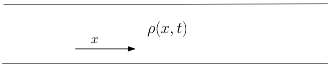


- How to model traffic flow on a stretch of highway ?
- Assumptions:
  - One lane
  - Continuum approximation

Macroscopic models of traffic



- How to model traffic flow on a stretch of highway ?
- Assumptions:
  - One lane
  - Continuum approximation
  - No ramp

Scalar macroscopic models

- Models of traffic based on mass conservation.

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0$$

Scalar macroscopic models

- Models of traffic based on mass conservation.

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0$$

- In integral form, with $N(t)$ the number of cars on $[0, L]$ at t, and $q(x, t)$ the flux of cars at (x, t):

$$\frac{\partial N}{\partial t}(t) = q(0, t) - q(L, t)$$

Scalar macroscopic models

- Models of traffic based on mass conservation.

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0$$

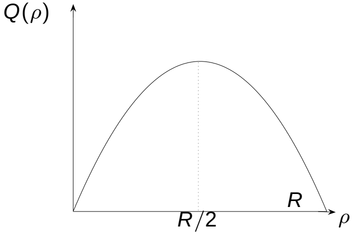
- Knowledge of the ‘fundamental diagram’, which is an empirical relation between the flux and the density.

The fundamental diagram

Classical fundamental diagrams include:

- Greenshields

$$q(\rho) = \rho V \left(1 - \frac{\rho}{R}\right)$$

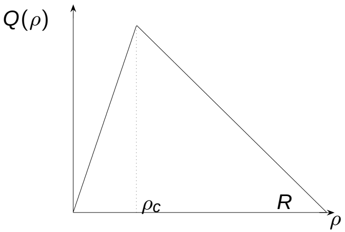


The fundamental diagram

Classical fundamental diagrams include:

- Newell-Daganzo

$$q(\rho) = \begin{cases} \rho V & \text{if } \rho \leq \rho_c \\ \frac{\rho V}{\rho_c - R}(\rho - R) & \text{if } \rho > \rho_c \end{cases}$$



The fundamental diagram

Classical fundamental diagrams include:

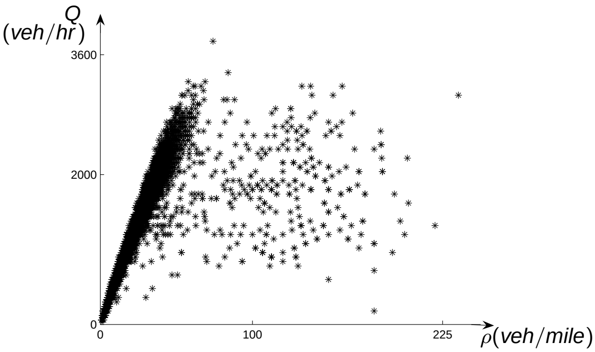
- Greenberg

$$q(\rho) = \rho\, v_0 \,\log \frac{\rho_{max}}{\rho}$$

- Papageorgiou

$$q(\rho) = \rho\, v_{max}\, \exp -\frac{1}{a} \left(\frac{\rho}{\rho_c}\right)^a$$

Real speed-density relation



- Has different behaviors in free-flow and congestion
- Is not single-valued in congestion

Model definition

- Two different modes for the dynamics

$$\begin{cases} \partial_t \rho + \partial_x(\rho\, v_f(\rho)) = 0 & \text{in free-flow} \\ \begin{cases} \partial_t \rho + \partial_x(\rho\, v_c(\rho, q)) = 0 \\ \partial_t q + \partial_x((q - q^*)\, v_c(\rho, q)) = 0 \end{cases} & \text{in congestion} \end{cases}$$

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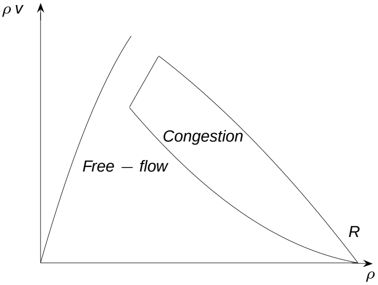
- A set-valued velocity function in congestion

$$v_f(\rho) = \left(1 - \frac{\rho}{R}\right) V \ \text{ and } \ v_c(\rho, q) = \left(1 - \frac{\rho}{R}\right) \frac{q}{\rho}$$

Improved features of this 2X2 model

- Characteristics speeds lower than vehicle speeds $\Rightarrow$ the model is anisotropic
- Vehicles stop only at maximal density

Limitations of the 2X2 phase transition model



But

- Solution of the discretized PDE constructed through wavefront-tracking is complex
- Model not customizable
- Free-flow speed is not constant

Motivation

The Colombo 2X2 phase transition model

A class of phase transition models

Model accuracy

A general 2X2 phase transition model

- Constant free-flow speed

$$v = v_f(\rho) := V \text{ in free-flow}$$

A general 2X2 phase transition model

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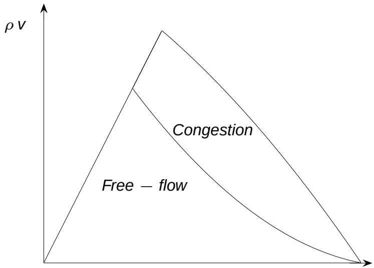
- Connection of the free-flow and congested phases



A general 2X2 phase transition model

- Constant free-flow speed

$$v = v_f(\rho) := V \;\text{ in free-flow}$$

- Connection of the free-flow and congested phases
- Introduction of the notion of perturbation, q

$$v = v_c(\rho, q) := v_c(\rho, 0)\,(1 + q) \;\text{ in congestion.}$$

$$\begin{cases} \partial_t \rho + \partial_x(\rho\, v) = 0 & \text{in free-flow } (\Omega_f) \\ \begin{cases} \partial_t \rho + \partial_x(\rho\, v) = 0 \\ \partial_t q + \partial_x(q\, v) = 0 \end{cases} & \text{in congestion } (\Omega_c) \end{cases}$$

Physical and mathematical constraints

- Positivity of speed

$$1 + q \geq 0$$

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- Strict hyperbolicity of the system in congestion

$$\forall\,(\rho, q) \in \Omega_c \quad \rho\,\partial_\rho v_c(\rho, 0) + q\,(v_c(\rho, 0) + \rho\,\partial_\rho v_c(\rho, 0)) \neq 0$$

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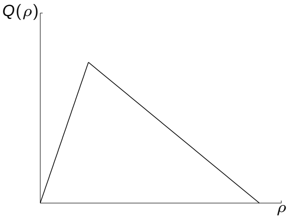
- Shape of Lax-curves

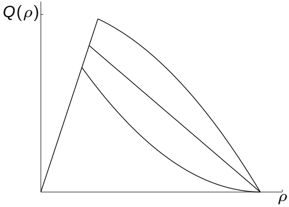
$$\rho\,(2\,\partial_\rho v_c(\rho, 0) + \rho\,\partial^2_{\rho\rho} v_c(\rho)) + q\,(2\,v_c(\rho, 0) + 4\,\rho\,\partial_\rho v_c(\rho, 0) + \rho^2\,\partial^2_{\rho\rho} v_c(\rho, 0))$$

is identically zero or has only one zero and is increasing.

Instantiation

- Newell-Daganzo

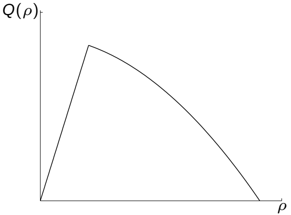


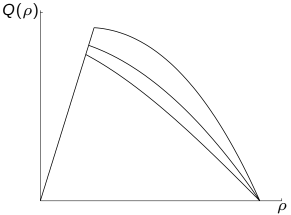


$$q \in [q_-, q_+] \text{ where } q_- \geq -1$$

Instantiation

- Greenshields

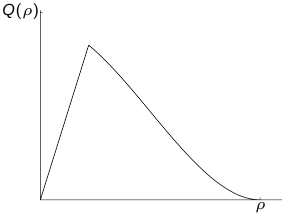


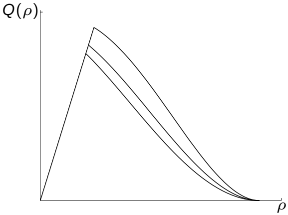


$$q \in [q_-, q_+] \text{ where } q_- \geq \frac{R}{2\,\rho_c - 3\,R}$$

Instantiation

- Non-concave flux





Bounds on both q_- and q_+

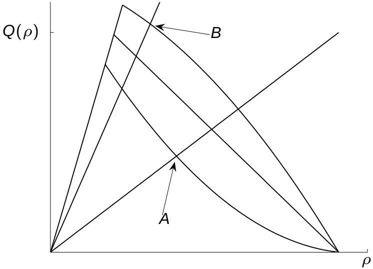
Specific features of the 2X2 phase transition model

- Modelization of different types of congestion
  - Wide moving traffic jams (backwards-moving jams $\Rightarrow$ usual first order congestion)
  - Synchronized traffic flow (forward-moving discontinuities with the same speed on both sides)
- Riemann problem solved by two different waves, one which has negative speed and one which has the speed of vehicles
- ‘Big shocks’ from classical LWR equation are phase transition in this framework
- Possibility of integrating both density measurements and speed measurements

Example

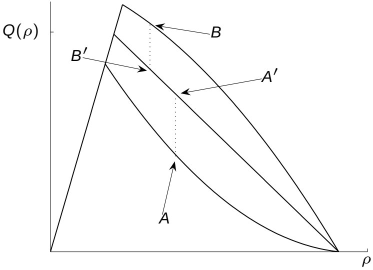
- Riemann problem with datum:

$$\begin{cases} u_{\text{left}} = (\rho_A, v_A) \\ u_{\text{right}} = (\rho_B, v_B) \end{cases}$$



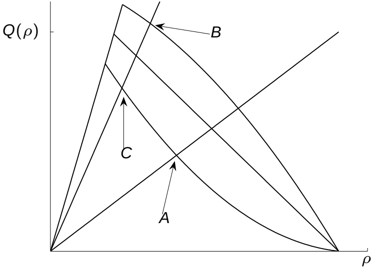
Example

- Backward-moving discontinuity (shockwave) between A' and B'



Example

- Backward-moving discontinuity (shockwave) between A and C followed by a forward moving discontinuity (contact discontinuity) between C and B



Dataset

- NGSIM dataset recorded on Highway 101 in Los Angeles
- Car trajectories available every 0.1 seconds
- 45 minutes long
- 0.4 miles

Comparison of triangular model and its extension

Error metric

$$E_u = \frac{\int_0^T \int_{x_0}^{x_1} \|u(t,x) - u_c(t,x)\|_1 dxdt}{\int_0^T \int_{x_0}^{x_1} \|u(t,x)\|_1 dxdt}$$

	E_ρ	E_v
Newell-Daganzo	17.8%	23.5%
N-D phase transition	15.0%	22.1%

Future work

- Comparison of travel-time estimations for different models on the Mobile Century dataset
- Define an optimal shape for the clouds of points in congestion

Questions ?