Using PDEs to Calculate Optimal Foot Motion for Walking

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Outline

• Research project background
• Approach for distributed system
  – Hybrid systems
• PDE model
  – Compass gait biped
• Implementation of control
  – Human motion data
• Refinements of model
Project Background

• Motivation
  – Quantitative Parkinson’s disease diagnosis

• Inertial measurement units for human movement analysis
  – Wireless Lagrangian sensors attached to feet
  – Triaxial accelerometers and gyroscopes
  – Gait analysis
Problem Statement

• Distributed system
  – Data collection along trajectory
  – Multiple parameters (position, velocity, time, etc.)

• PDE to calculate optimal foot motion to take a step during walking
Approach

- Walking: continuous and discrete dynamics
  - Hybrid systems
  - State consists of vector signals
  - Flow equation $f(x)$ and jump equation $g(x)$
Hybrid Systems

• Simple hybrid system: a 4-tuple

\[ H = (D, G, R, f) \]

where

\[ D \subseteq \mathbb{R}^n \] domain
\[ G \subseteq D \] guard
\[ R : G \to D \] reset map, from impact equations
\[ f \] vector field on D, i.e. \( \dot{x} = f(x) \)

• Simple hybrid control system: a 6-tuple

\[ H = (D, U, G, R, f, g) \]

where

\[ D, G, R \] domain, guard, reset map
\[ U \subseteq \mathbb{R}^k \] set of admissable controls
\[ (f, g) \] control system, i.e. \( \dot{x} = f(x) + g(x)u \)
Compass Gait Biped Model

- From robotics and control
- Parameters of model

\[ m, m_H \] masses of limbs, hip
\[ a, b \] length of limbs
\[ \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \] angle vector
\[ u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \] control torque vector
Langrangian Dynamics

• Langrangian of compass gait biped model

\[ L(\theta, \dot{\theta}) = \frac{1}{2} \dot{\theta}^T \dot{\theta} \]
where

\[ M(\theta) \quad \text{inertial matrix, i.e. } \frac{1}{2} \dot{\theta}^T M(\theta) \dot{\theta} \quad \text{kinetic energy} \]

\[ P(\theta) \quad \text{potential energy} \]

• Controlled Euler-Lagrange equations

\[ M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = Su \]
where

\[ M(\theta) \quad \text{inertial matrix} \]

\[ C(\theta, \dot{\theta}) \quad \text{coriolis matrix} \]

\[ G(\theta) = \frac{\partial P}{\partial \theta}(\theta) \]
constraint function $V(\theta) = \cos \theta_1 - \cos \theta_2$

SHMCS a 6-tuple

$H = (D, U, G, R, f, g)$

where

$$D = \left\{ \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \in \mathbb{R}^4 : V(\theta) \geq 0 \right\}$$

$$G = \left\{ \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \in \mathbb{R}^4 : V(\theta) = 0, \left[ \frac{\partial V}{\partial \theta}(\theta) \right]^T \dot{\theta} < 0 \right\}$$

$R$ from kinematic constraint function

$U = \mathbb{R}^2$

$$f(\theta, \dot{\theta}) = \begin{bmatrix} \dot{\theta} \\ M(\theta)^{-1}(-C(\theta, \dot{\theta})\dot{\theta} - G(\theta)) \end{bmatrix}$$

$$g(\theta, \dot{\theta}) = \begin{bmatrix} 0_{2 \times 2} \\ M(\theta)^{-1} S \end{bmatrix}$$
Implementation of Control: $\theta$ and $\dot{\theta}$

- Experimental human motion data (five cycles)

![Graphs showing angular positions and velocities for Non-Parkinson's and Parkinson's cases.](image-url)
Comparison with Biped Model: $\theta$ and $\dot{\theta}$

- **One cycle** *(reference for left figures: Asano, 2004)*

### Biped Model

- Angular positions ($\theta_1$, $\theta_2$) vs. time (0-0.8 s)
- Angular velocities ($\dot{\theta}_1$, $\dot{\theta}_2$) vs. time (0-0.8 s)

### Non-Parkinson’s

- Angular positions ($\theta_1$, $\theta_2$) vs. time (0.8-1.2 s)
- Angular velocities ($\dot{\theta}_1$, $\dot{\theta}_2$) vs. time (0.8-1.2 s)
Comparison with Biped Model: $\theta$ and $\dot{\theta}$

- One cycle (reference for left figures: Asano, 2004)

Biped Model

Non-Parkinson’s

Parkinson’s

Comparison with Biped Model: $\theta$ and $\dot{\theta}$
Implementation of Control: $u$ and $L$

- Experimental human motion data (five cycles)

Non-Parkinson’s

Parkinson’s

Control inputs (Nm)

Lagrangian (J)
Comparison with Biped Model: $u$ and $L$

- **Four cycles** (reference for left figures: Asano, 2004)

Biped Model

Non-Parkinson’s
Comparison with Biped Model: $u$ and $L$

- Four cycles (reference for left figures: Asano, 2004)

Biped Model

Non-Parkinson’s

Parkinson’s
Refinements of Model

• Assumptions of biped model:
  – Impacts are perfectly inelastic (no bounce)
  – No slipping of the stance leg at ground contact
  – Transfer of support between swing and stance legs is instantaneous, i.e. negligible double support phase
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Summary

• Research project background
  – Quantitative diagnosis of Parkinson’s disease

• Approach for distributed system
  – Simple hybrid mechanical control system

• PDE model
  – Compass gait biped

• Implementation of control
  – Human motion data for non-Parkinson’s and Parkinson’s subjects

• Refinements of model
  – Incorporate double support phase
References

- Spong, M. W., “Passivity based control of the compass gait biped,” in *Proc. of IFAC World Congress*, Beijing, China, 1999.