Adjoint-Based Electromagnetic Shape Optimization

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EE 291

How to Represent Shapes

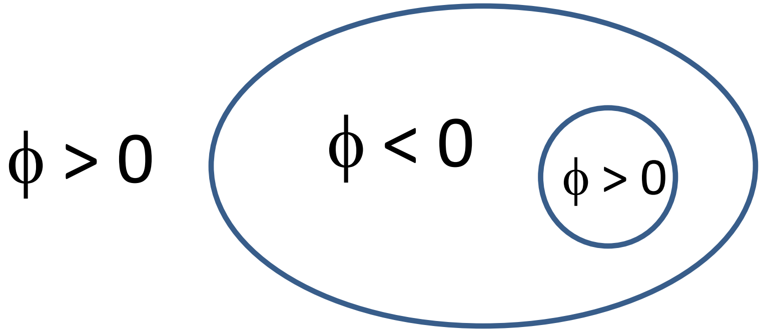
- Boundaries determine the shape

$$\partial\Omega(t) = \{(\vec{x}, t) \mid \varphi(\vec{x}, t) = 0\}$$

- Rather than parametrizing boundary, more effective to embed boundary as the zero level set of a function, which we call ϕ

Interior: $\phi < 0$

Exterior: $\phi > 0$



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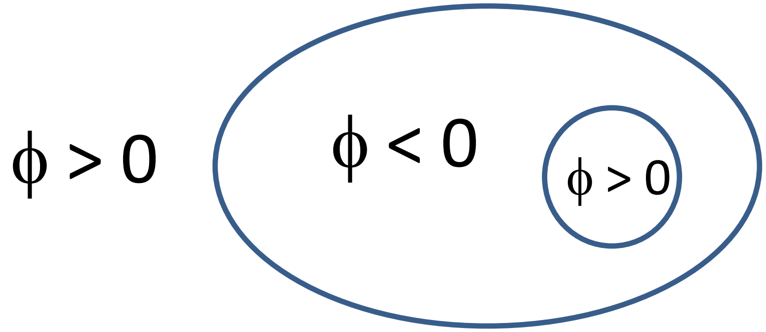


To move boundary,

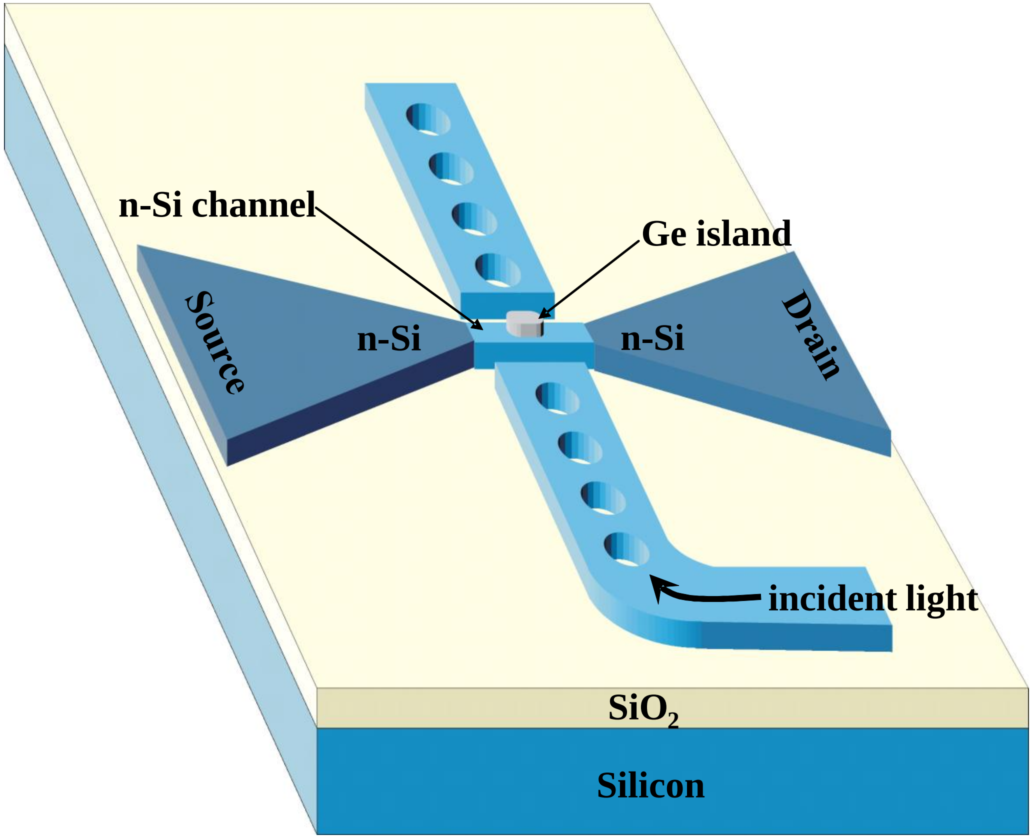
$$\frac{d}{dt}\varphi(\vec{x}, t) = \frac{\partial \varphi}{\partial t} + \frac{d\vec{x}}{dt} \bullet \nabla \varphi = 0$$

Hamilton-Jacobi Equation

$$\vec{V} = \frac{d\vec{x}}{dt}$$

Velocity determines boundary movement!

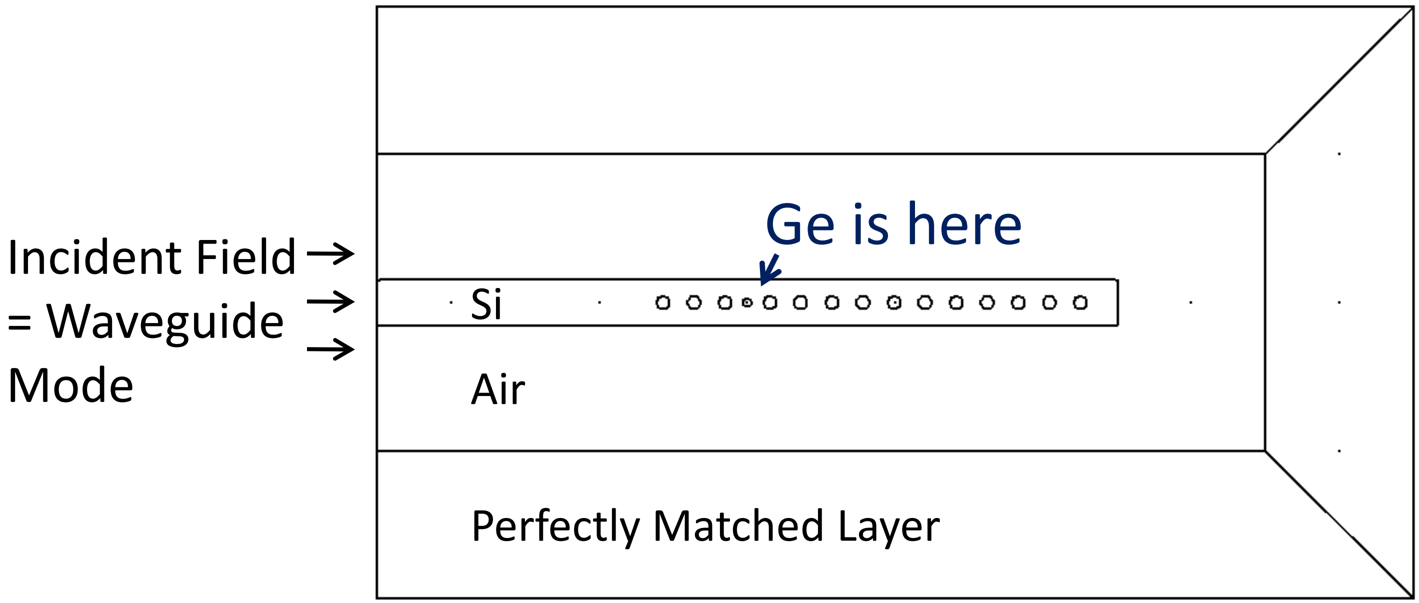
Application: Photodetector Design



- Ge very effective absorber
- Small channel needed for SNR, bandwidth

→ Need highly efficient waveguide!

Waveguide Simulation



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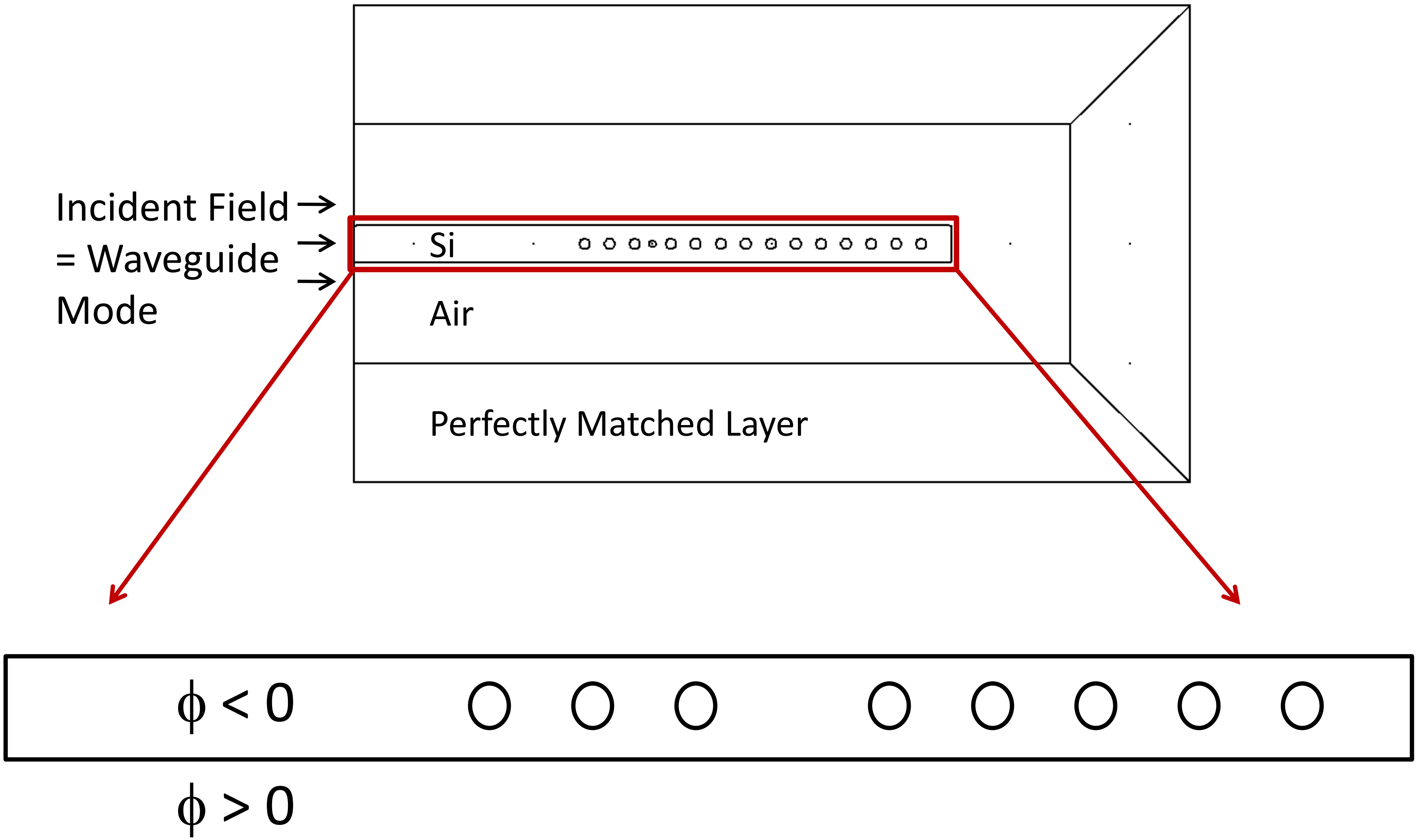
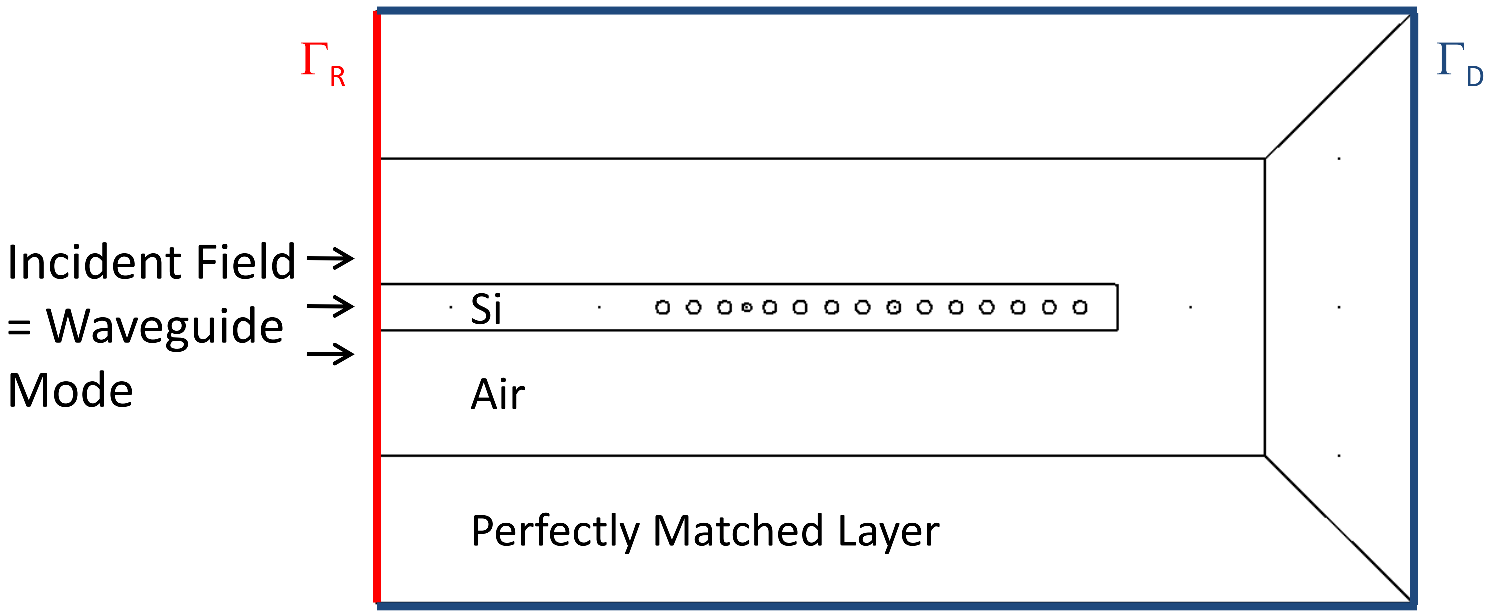


This is our design area

Waveguide Simulation



$$\left(\nabla^2 + \varepsilon(x,\phi)\frac{\omega^2}{c^2}\right)u = 0$$ throughout domain

$$\frac{\partial u}{\partial n} + au = b$$ on Γ_R

$$u = 0$$ on Γ_D

Weak Form of Maxwell's Eqn.'s

$$\nabla^2 u + \varepsilon(x,\phi)\frac{\omega^2}{c^2}u = F$$

$$\int_A \left(\bar{v}\nabla^2 u + \varepsilon(x,\phi)\frac{\omega^2}{c^2}\bar{v}u \right) dA = \int_A \bar{v}F dA$$ Inner product with test function v

$$\int_A \left(-\nabla\bar{v}\bullet\nabla u + \varepsilon(x,\phi)\frac{\omega^2}{c^2}\bar{v}u \right) dA + \int_{\partial A} \bar{v}\frac{\partial u}{\partial n} ds = \int_A \bar{v}F dA$$ Integration by parts

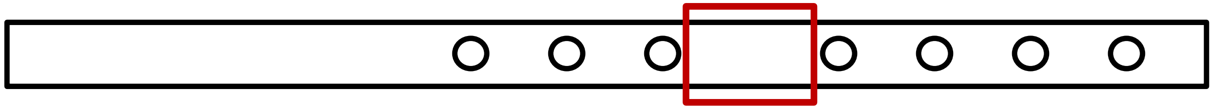
$$\underbrace{\int_A \left(-\nabla\bar{v}\bullet\nabla u + \varepsilon(x,\phi)\frac{\omega^2}{c^2}\bar{v}u \right) dA - a\int_{\Gamma_R} \bar{v}u ds}_{a(u,v)} + \underbrace{b\int_{\Gamma_R} \bar{v} ds = \int_A \bar{v}F dA}_{L(v)}$$ Insert b.c.'s

$$a(u,v) = L(v) \qquad \forall v \in V$$

*Note: $a(u,v) = a(\bar{v},\bar{u})$

Shape Derivatives

Consider the merit function $J = \int_D |u|^2 dA = \int_D u\bar{u}\, dA$



$$\delta J = \frac{\partial J}{\partial \varphi}\delta\varphi + \frac{\partial J}{\partial u}\delta u + \frac{\partial J}{\partial \bar{u}}\delta\bar{u} = 2\,\mathrm{Re}\int_D \bar{u}\,\delta u\, dA$$

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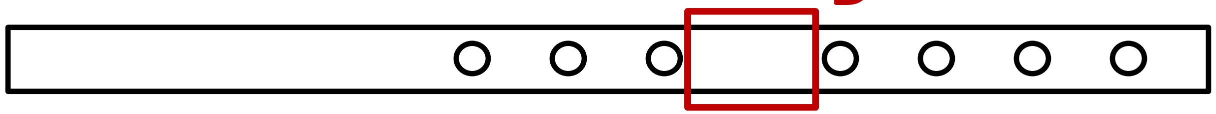

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Define an "adjoint" field w such that

$$\nabla^2 w + \varepsilon(x,\phi)\frac{\omega^2}{c^2} w = 2\bar{u}\chi_D \qquad \chi_D = \begin{cases} 1 & x \in D \\ 0 & x \notin D \end{cases}$$

$$\frac{\partial u}{\partial n} + au = 0 \qquad \text{on } \Gamma_R$$

Weak form of u: $a(u,v) = -b \int_{\Gamma_R} \bar{v} ds$

Take differential: $a(\delta u,v) = -\frac{\omega^2}{c^2} \int_A \bar{v} u \frac{\partial \varepsilon}{\partial \varphi} \delta\varphi \;\; dA$ $\quad v \to \bar{w}$

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Weak form of w: $a(w,v) = 2\int_D \bar{v}\bar{u}\, dA$ $\qquad v \rightarrow \delta\bar{u}$

$$a(\bar{w}, \delta\bar{u}) = a(\delta u, w) \quad \longrightarrow \quad 2\int_D \bar{u}\,\delta u\, dA = -\frac{\omega^2}{c^2}\int_A u w \frac{\partial \varepsilon}{\partial \varphi}\delta\varphi \; dA$$

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$$\delta\varphi = -V_n\,|\nabla\varphi|\,dt$$

Hamilton-Jacobi

$$\varepsilon(\varphi) = \varepsilon_{Si}H(-\varphi) + \varepsilon_{Air}H(\varphi) \quad \longrightarrow \quad \frac{\partial \varepsilon}{\partial \varphi} = -(\varepsilon_{Si} - \varepsilon_{Air})\delta(\varphi)$$

Helmholtz Shape Derivative

Putting it all together,

$$\delta J = -\frac{\omega^2}{c^2}(\varepsilon_{Si} - \varepsilon_{Air})dt \operatorname{Re} \int_A uwV_n |\nabla \varphi| \delta(\varphi) dA$$

$$\delta J = -\frac{\omega^2}{c^2}(\varepsilon_{Si} - \varepsilon_{Air})dt \operatorname{Re} \int_{\partial\Omega} uwV_n dA$$

We can choose $V_n = -\overline{u}\,\overline{w}$ and we have an ascent algorithm!

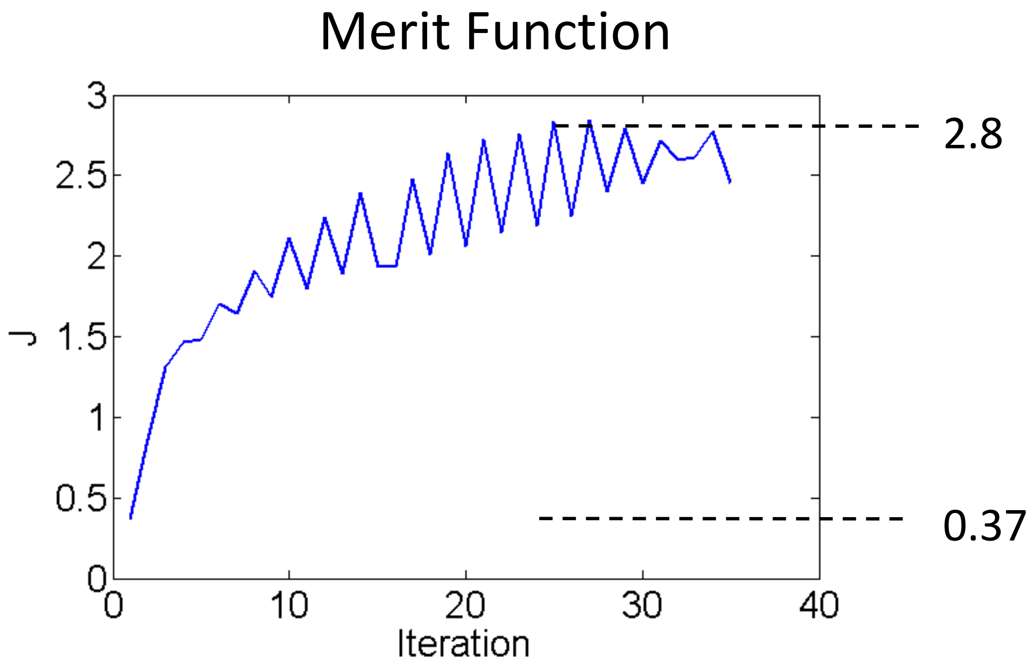
$$\delta J > 0$$

Shape Optimization Process

Use Comsol – standard FEM software

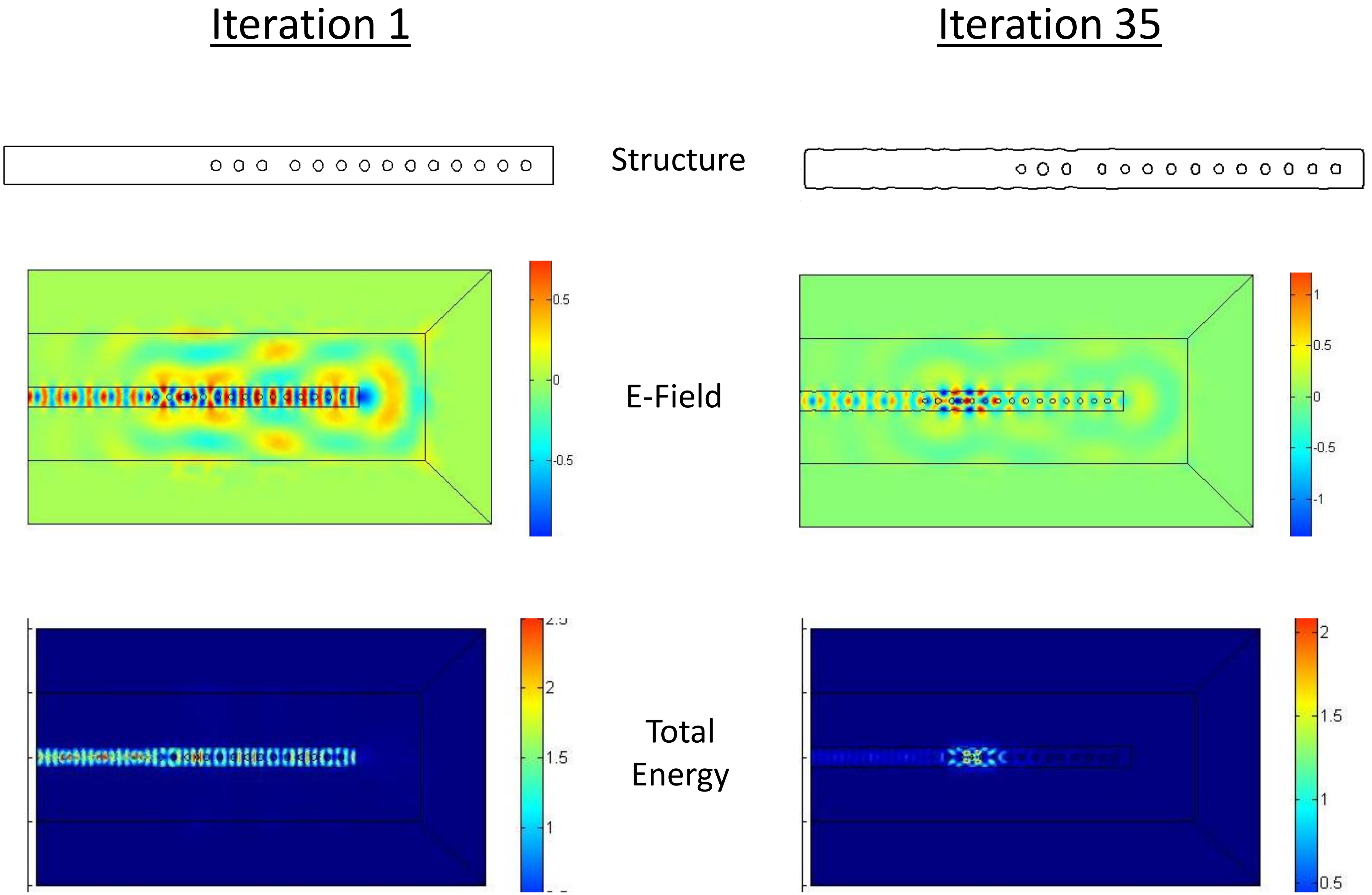
- Define geometry in Matlab
- Pass geometry to Comsol
- Solve for u, w in Comsol
- Use u, w to define velocity
- Update geometry in Matlab w/ Hamilton-Jacobi
- Loop

Test Run



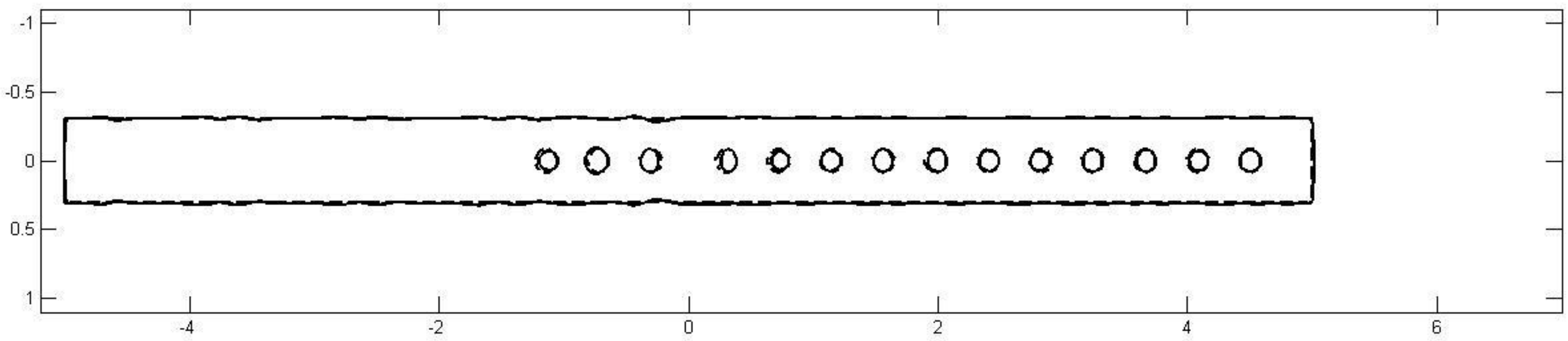
For many steps, restricted step size to ensure $J_{new} > 0.75*J_{old}$

Factor of 8 improvement in merit function!



Iteration 1
Iteration 35
Structure
E-Field
Total Energy
0.5
0
-0.5
2
1.5
1
1
0.5
0
-0.5
-1
2
1.5
1
0.5

Comparison of Initial and Final Iterations



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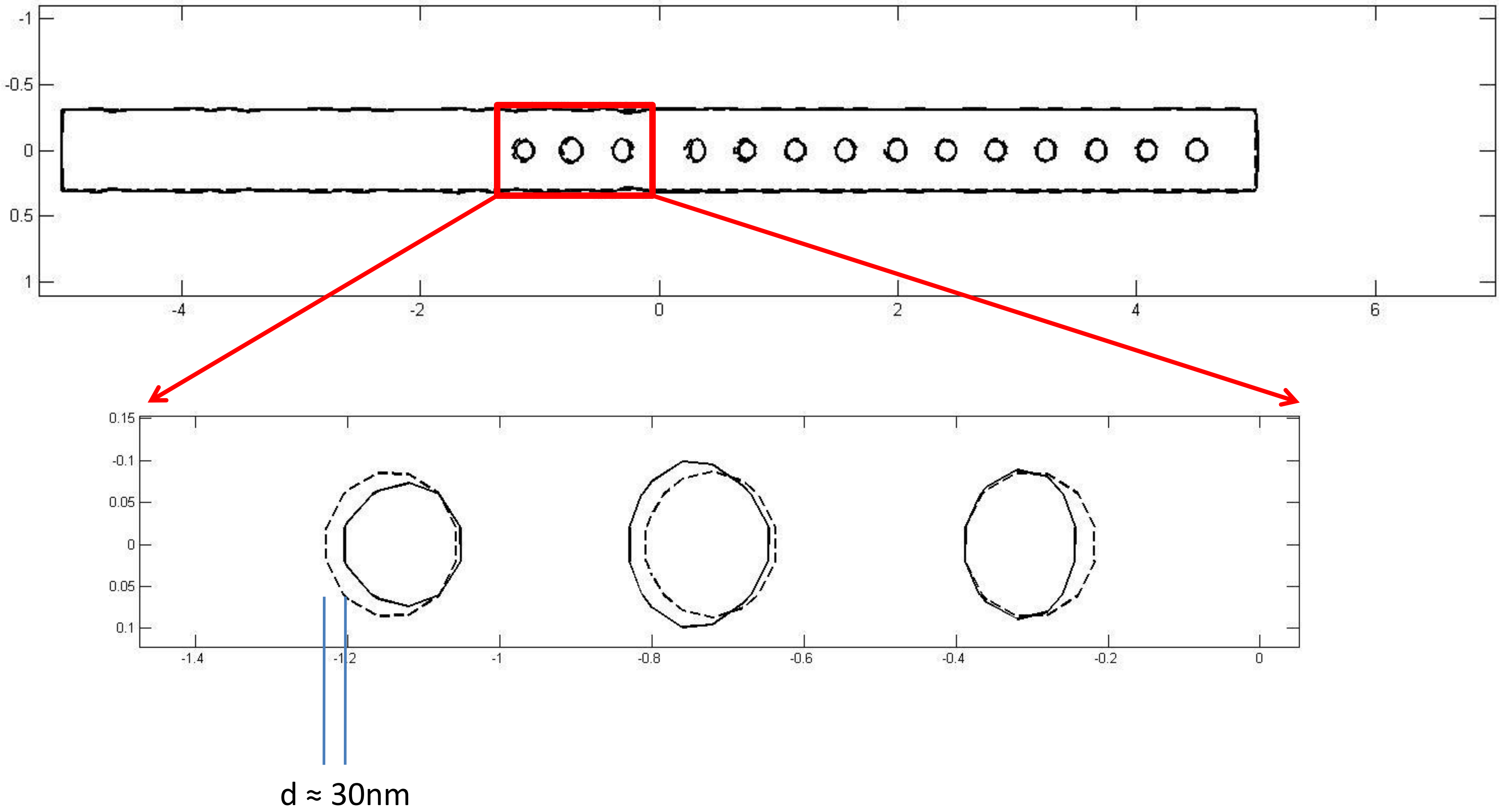

Timing

35 iterations took 2 hours
- 75 min for simulations
- 45 min for everything else (creating/updating structures, passing data back and forth, etc.)

Because of 75 % restriction, 35 iterations was really > 100 runs