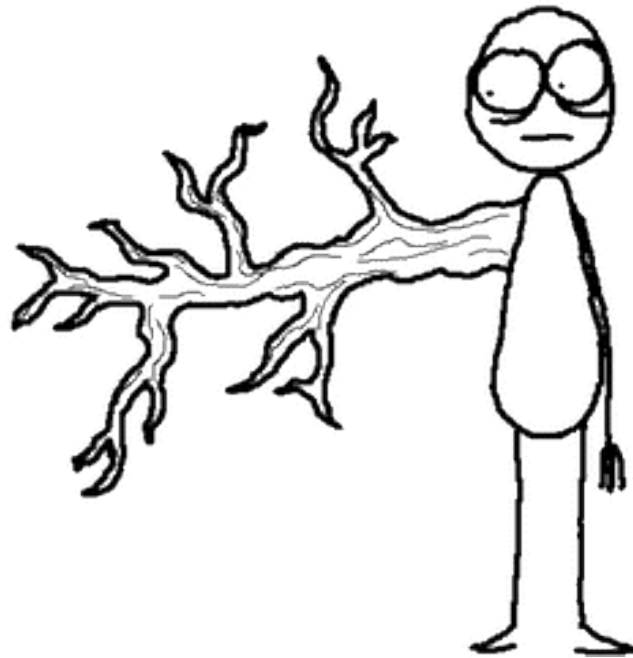




ME 236: Final Project

# Numerical Growth & Branching



Presented by  
Timothy Tresieras





# Background / Motivation

**Time Lapse Plant Growth  
Demonstration on YouTube**



# Background / Motivation

## Time Lapse Plant Growth Demonstration on YouTube

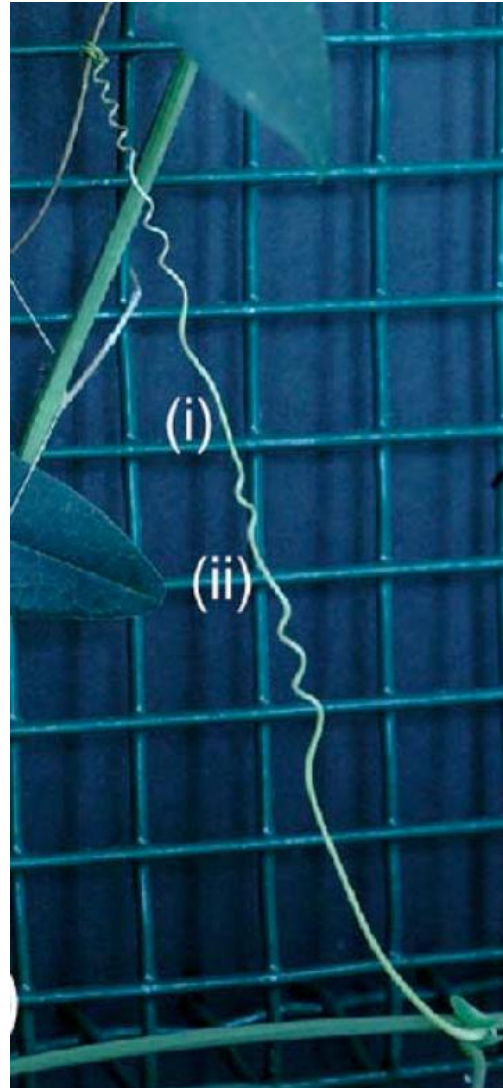
### Observations

- Movement effects other branches
- Lateral Growth / Tip Growth
- Material properties change in time
- Reactionary vs. Evolutionary time scale



# Background / Motivation

Another phenomenon  
we would like to capture.





# Background / Motivation

When one end is cut its  
general shape is preserved.





# Background / Motivation

When one end is cut its  
general shape is preserved.



Drying out a rose.



# Background / Motivation

## Adapted Rod Theory: Integral Balance Laws

$$\frac{d}{dt} \int_{\xi_1}^{\xi_2} \rho_0 \dot{\mathbf{r}} d\xi - [\rho_0 \dot{\mathbf{r}} \dot{\xi}]_{\xi_1}^{\xi_2} - \int_{\xi_1}^{\xi_2} \frac{m}{\rho_0} \dot{\mathbf{r}} d\xi = [\mathbf{n}]_{\xi_1}^{\xi_2} + \int_{\xi_1}^{\xi_2} \rho_0 \mathbf{f} d\xi + \int_{\xi_1}^{\xi_2} \mathbf{F} \delta(\xi - \gamma) d\xi$$

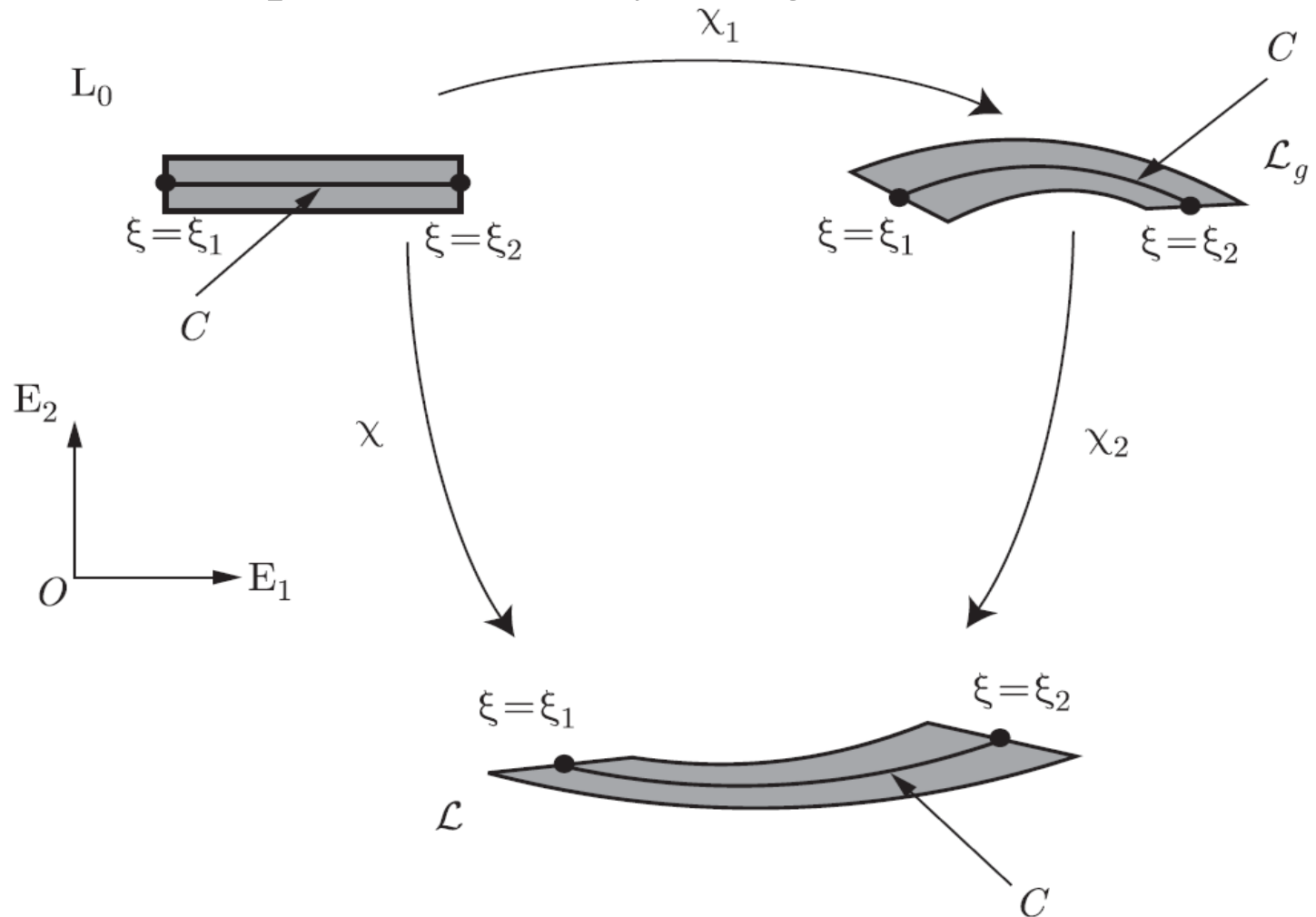
Balance of Linear Momentum

\*Additional Balance Laws of Similar Form



# Background / Motivation

## Adapted Rod Theory: Integral Balance Laws







# Background / Motivation

Adapted Rod Theory: Local Balance Laws

$$\rho_g \ddot{\mathbf{r}} = \rho_g \mathbf{f} + \frac{\partial \mathbf{n}}{\partial s_g}$$

$$\rho_g I \ddot{\theta} \mathbf{E}_3 = \rho_g \mathbf{m}_a + \frac{\partial \mathbf{r}}{\partial s_g} \times \mathbf{n} + \frac{\partial \mathbf{m}}{\partial s_g}$$

Equations of Motion

$$\llbracket \mathbf{n} + \dot{\gamma} \rho_0 \dot{\mathbf{r}} \rrbracket + \mathbf{F} = \mathbf{0}$$

$$\llbracket \mathbf{m} + \dot{\gamma} \rho_0 I \dot{\theta} \mathbf{E}_3 \rrbracket + \mathbf{M} = \mathbf{0}$$

Jump Conditions



# Background / Motivation

Adapted Rod Theory: Local Balance Laws

$$\rho \mathbf{f} + \mathbf{n}' = \mathbf{0}$$

$$\mathbf{m}' + \mathbf{r}' \times \mathbf{n} = \mathbf{0}$$

Equations of Motion

$$[[\mathbf{n}]] + \mathbf{F} = \mathbf{0}$$

$$[[\mathbf{r} \times \mathbf{n} + \mathbf{m}]] + \mathbf{M} = \mathbf{0}$$

Jump Conditions



# Simplified Equations of Motion

## Assumptions

- Equilibrium (i.e. ODE's)
- Simple constitutive equation

$$\mathbf{m} = EI \left( \frac{\partial \theta}{\partial s} - \frac{\partial \theta_g}{\partial s} \right) \mathbf{E}_3 = EI(\nu - \nu_g) \mathbf{E}_3$$

- Body Force = Gravity
- Homogeneous / Constant Cross-Section

(This assumption is later relaxed)



# Simplified Equations of Motion

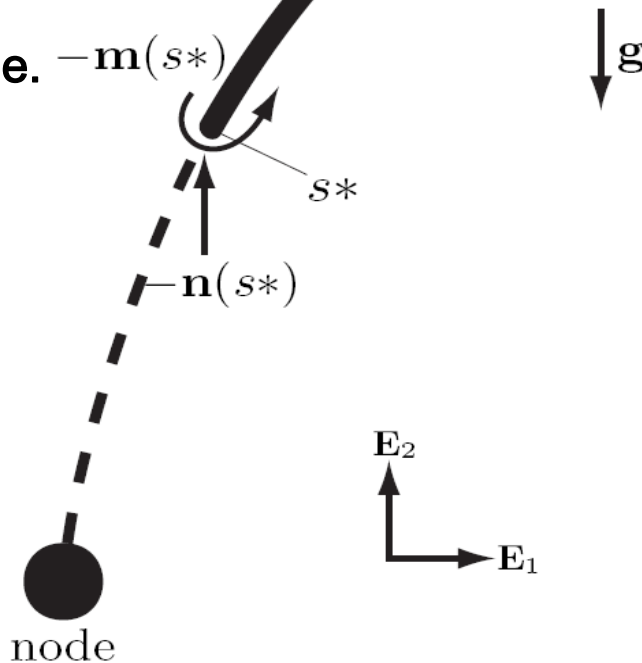
$$\theta' = \nu_r + \nu_g$$

$$\nu_r' = \frac{1}{EI} (\rho g (L - s) + n_l) \cos(\theta)$$

} Reduced Order

where  $\nu_r = \nu - \nu_g$

\*Stiffness parameter may vary in time.





# Branching



# Dimensionless Equations of Motion

## Assumptions

- Only self-weight, i.e.,  $n_1=0$
- No intrinsic curvature
- Free Tip => Boundary Condition:  $\nu(L)=0$
- No Growth / No Evolution

$$\theta' = \hat{\nu}$$

$$\hat{\nu}' = \alpha(1 - \hat{s}) \cos(\theta)$$

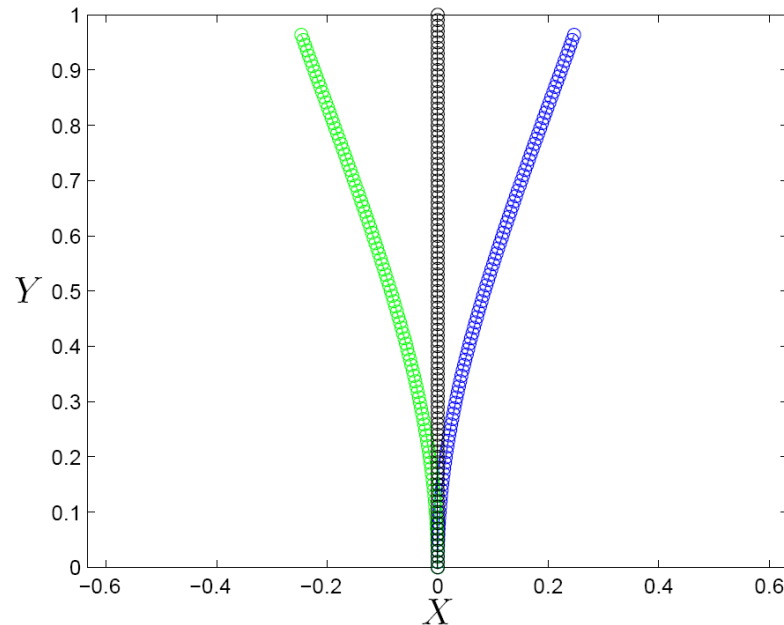
$$\text{where } \alpha = \frac{\rho_0 g}{EI} L^3, \quad \hat{\nu} = \nu L, \quad \hat{s} = \frac{s}{L}$$



# Branching Experiments

## Problems

- BVP B.C.'s
- Multiple Solutions / No Solution
- B.C.'s Relation to Stable & Unstable Configurations



**Fig. 1** When  $\alpha = 8$  the inverted branch, the black configuration of unit length, is unstable and will fall to one of two stable configurations, the green or blue configuration, if slightly perturbed. Thus, a BVP with boundary conditions of  $\nu_{nd}(L) = 0$  and  $\theta(0) = 90^\circ$  will have three solutions, two stable and one unstable.

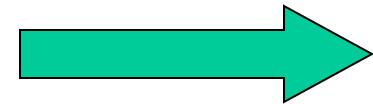


# Branching Experiments

(New Approach)

## Avoid BVP

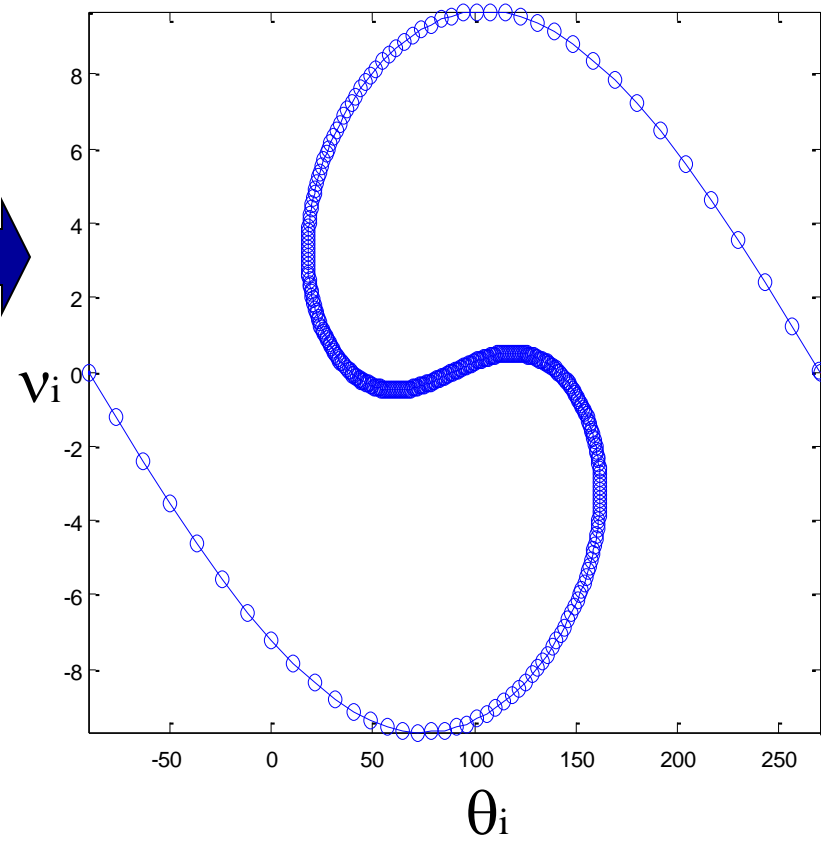
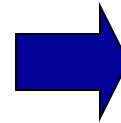
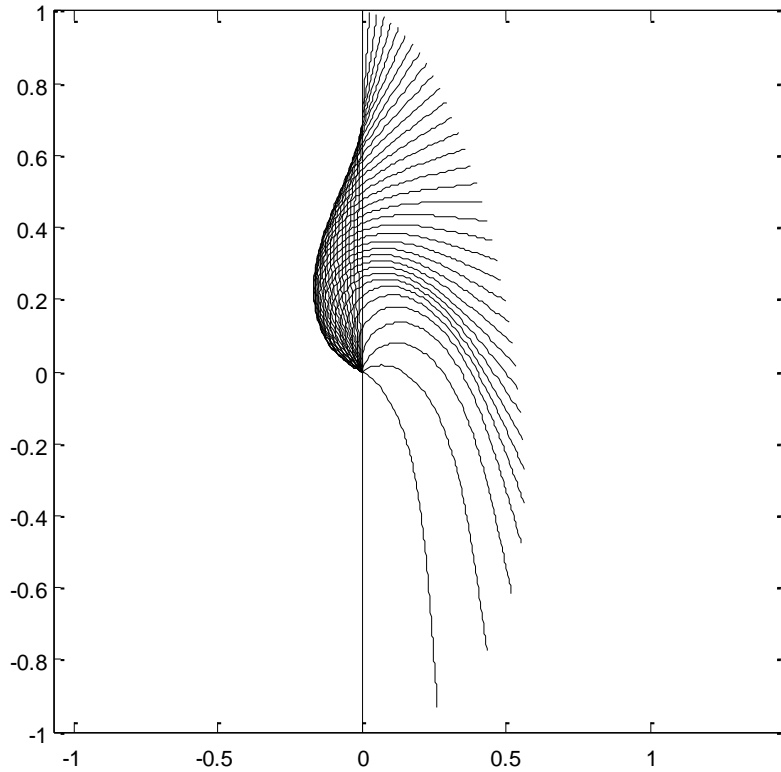
- Integrate Backwards as an IVP
- Start from  $(\theta_f, v_f)$
- Vary  $\theta_f$  at small (equal) intervals between 0 to 360 degrees
- Use data to produce an S-Curve





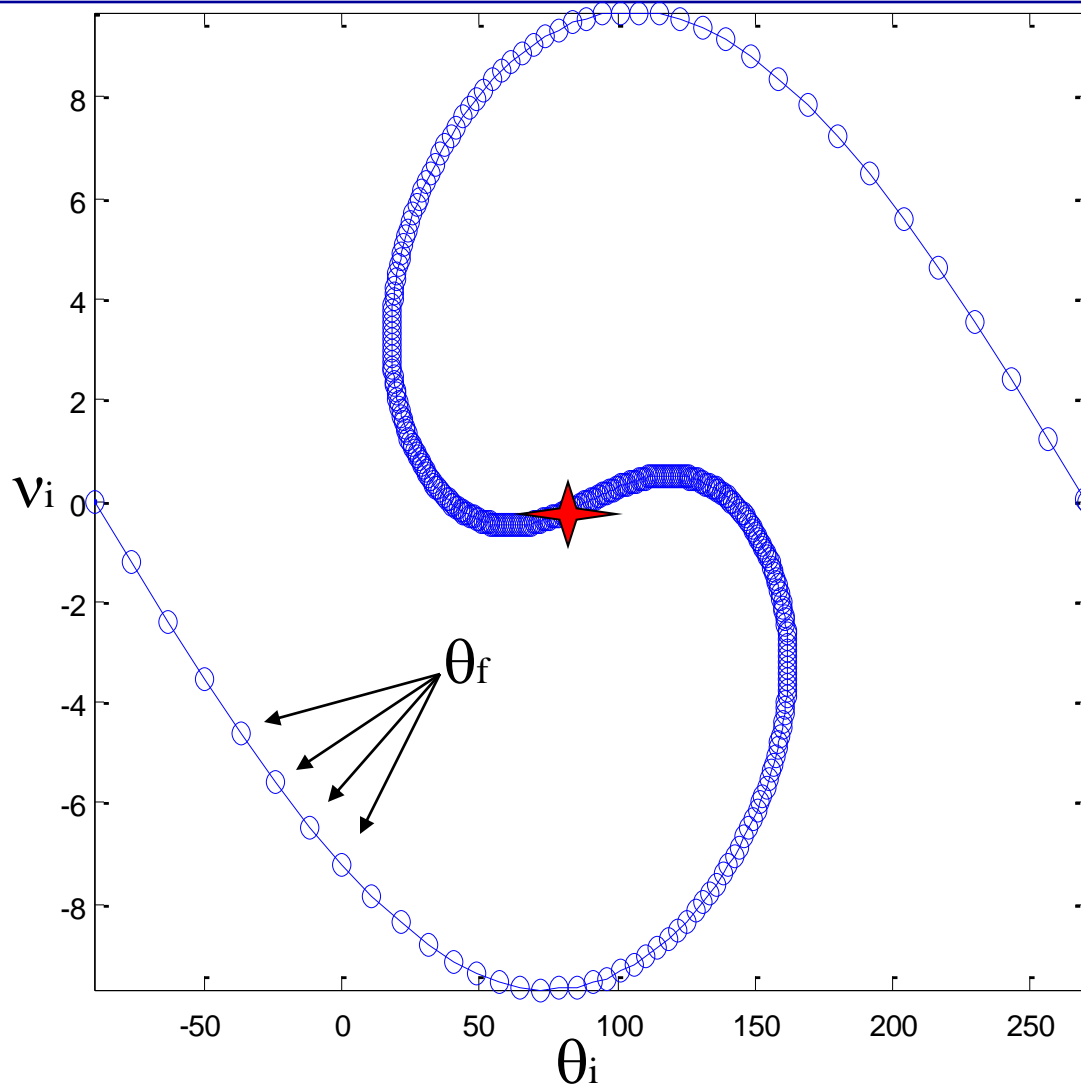


# The "S"-curve





# The “S”-curve

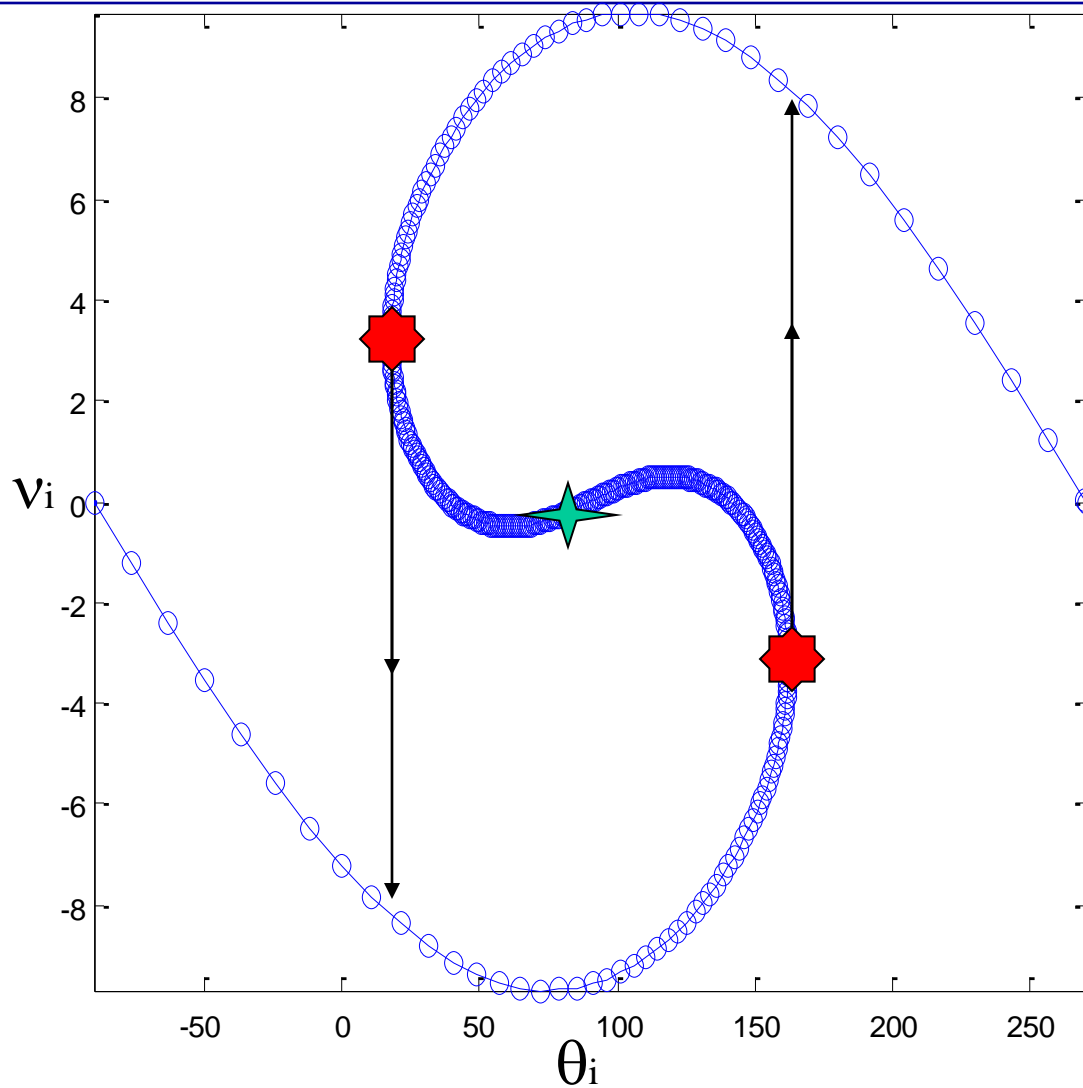


- Non-dim  $\alpha=30$  (flexible)
- Curve parameterized by  $\theta_f$
- Symmetry due to no intrinsic curvature
- Stable and Unstable configurations

 Inverted Column



# The “S”-curve

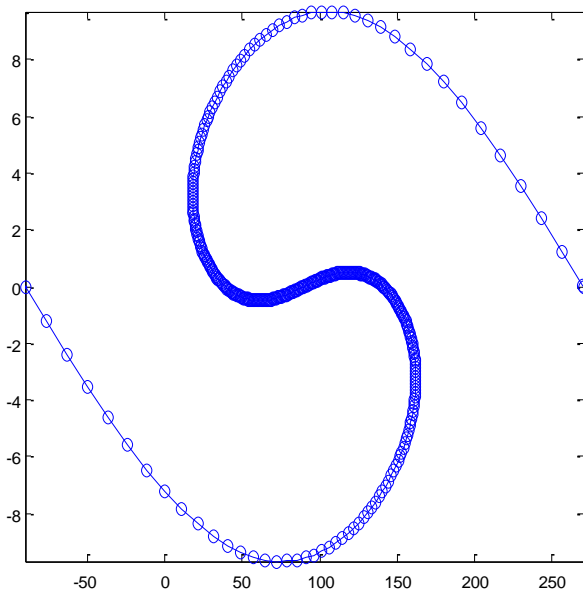


 Jumping Point

\*The unstable configurations lie between these points on the curve.



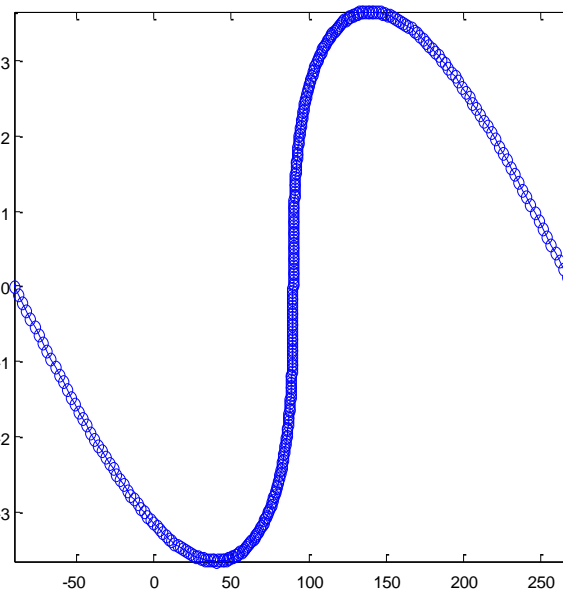
# The “S”-curve



- Non-dim Compliance = 30 (Flexible)

- Stretched Curve

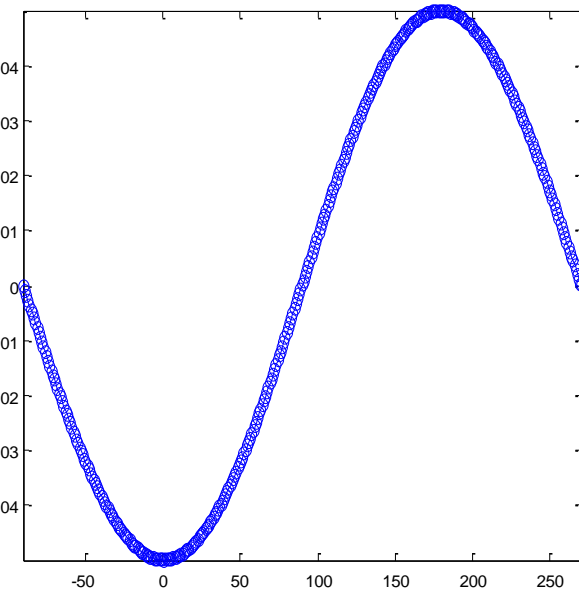
- Unstable configurations



- Non dim Compliance = 7.845 (Bifurcation Point)

- Stretched Curve

- Critically Stable Column



- Non-dim Compliance = 0.1 (Rigid)

- Unstretched Curve

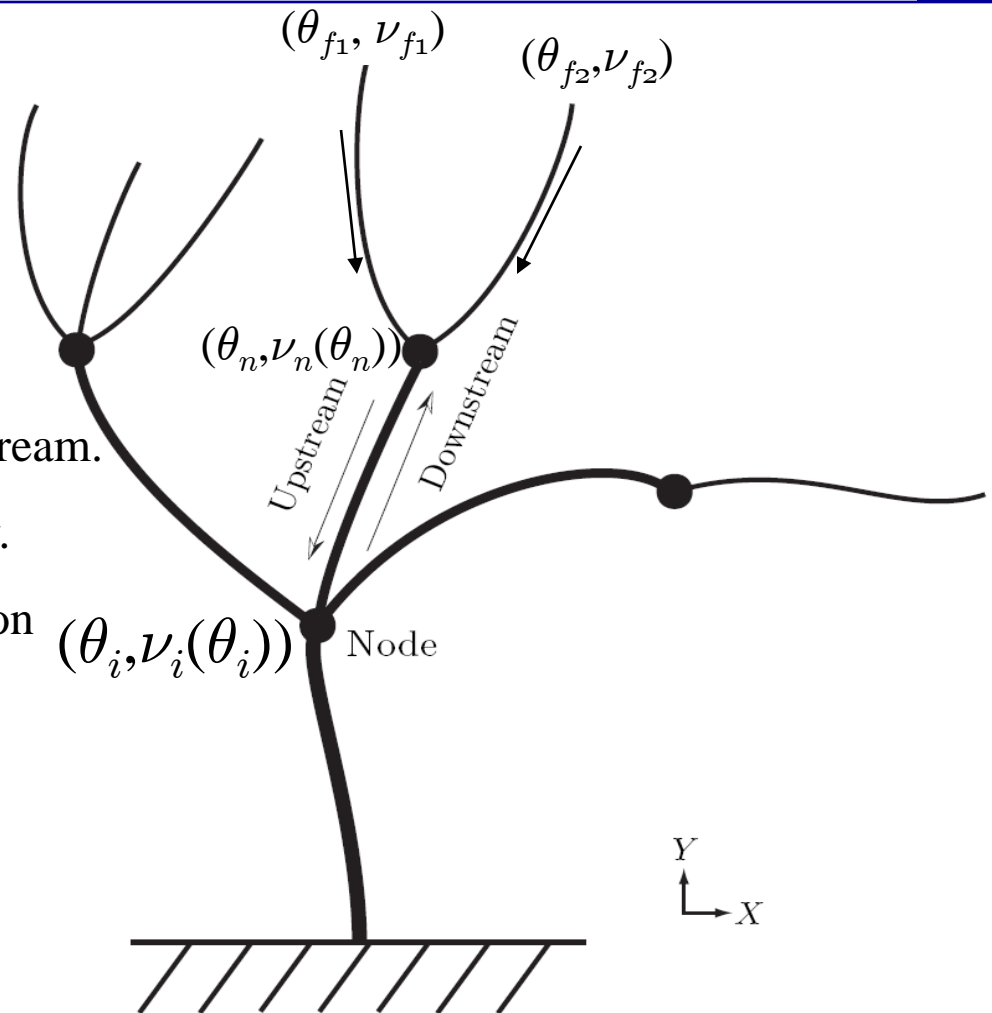
- All stable configurations

- Approx  $M = mgl \cdot \cos(\theta)$



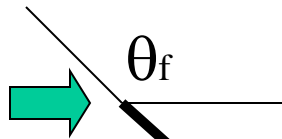
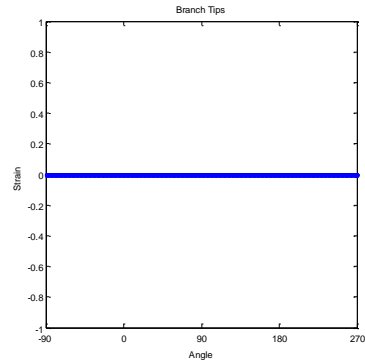
# Numerical Branching Method

- Values for  $\nu_f$  at the tip are prescribed (e.g. Zero for free branch tips)
- Information from S-curves goes upstream.
- Plant structure has a contractive flow.
- The nodes conforms to jump condition constraints





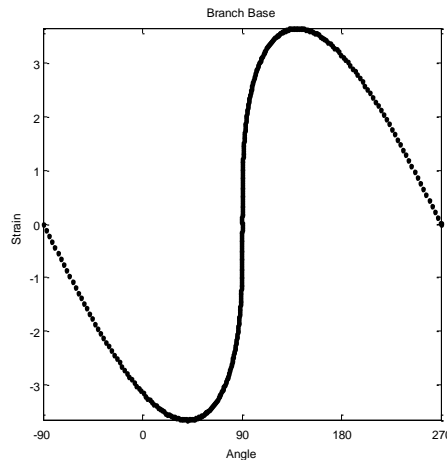
# Numerical Branching Method



$\theta_f$

Free Tip Branch

Strain as a function of  $\theta_f$

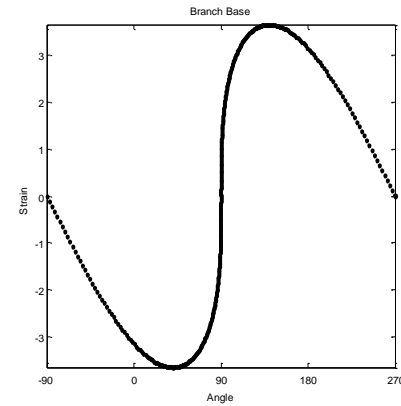


$\theta_i$

Strain as a function of  $\theta_i$



# Numerical Branching Method

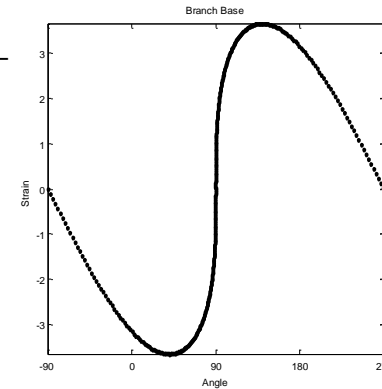


S-Curve from Branch 1

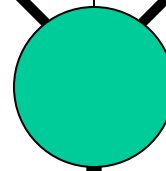
Offset Angle: CCW=>+

$\theta_+$

$\theta_-$



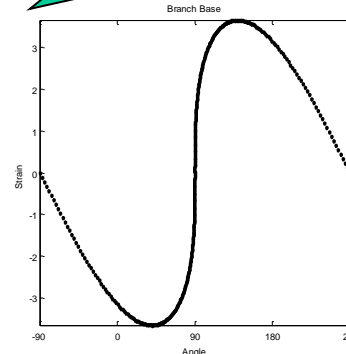
S-Curve from Branch 2



Node

OUTPUT

Calculated Weight of  
Upstream Branches



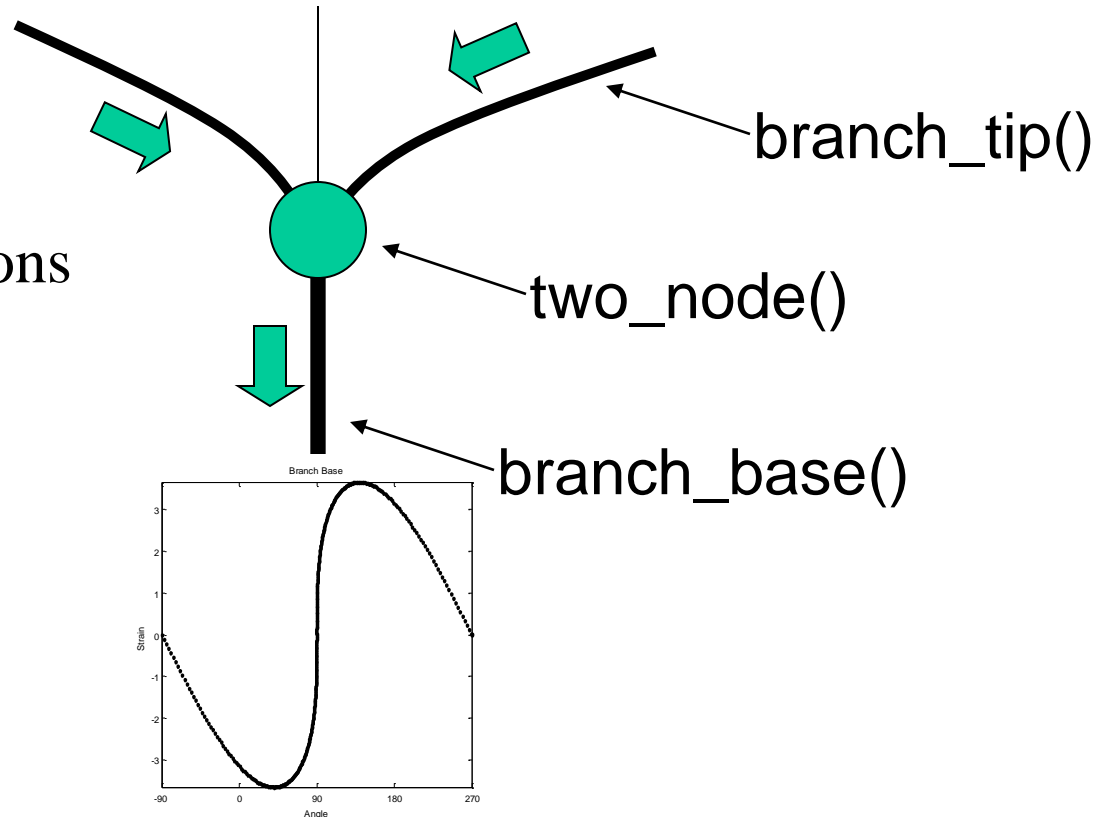
OUTPUT

Linearly Weighted  
Combination of S-Curve  
1&2 sent to Base Branch



# Numerical Branching Method

Representative  
MatLab Functions



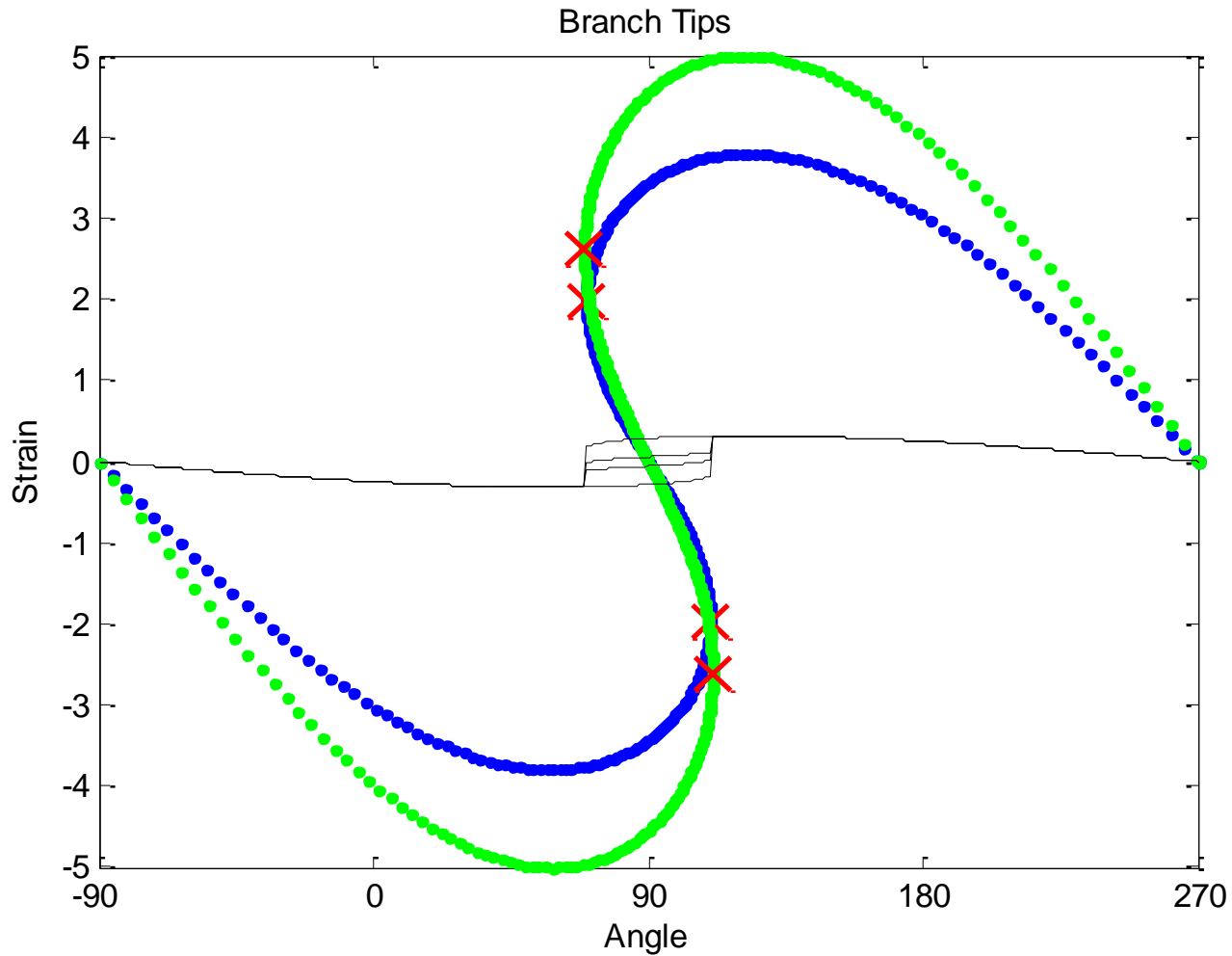
Initial Inputs for Each Branch: Weight ( $\rho g$ ); Length ( $L$ ); Stiffness ( $EI$ );

Offset Angle; Resolution; Plotting Parameters





# Numerical Branching Method

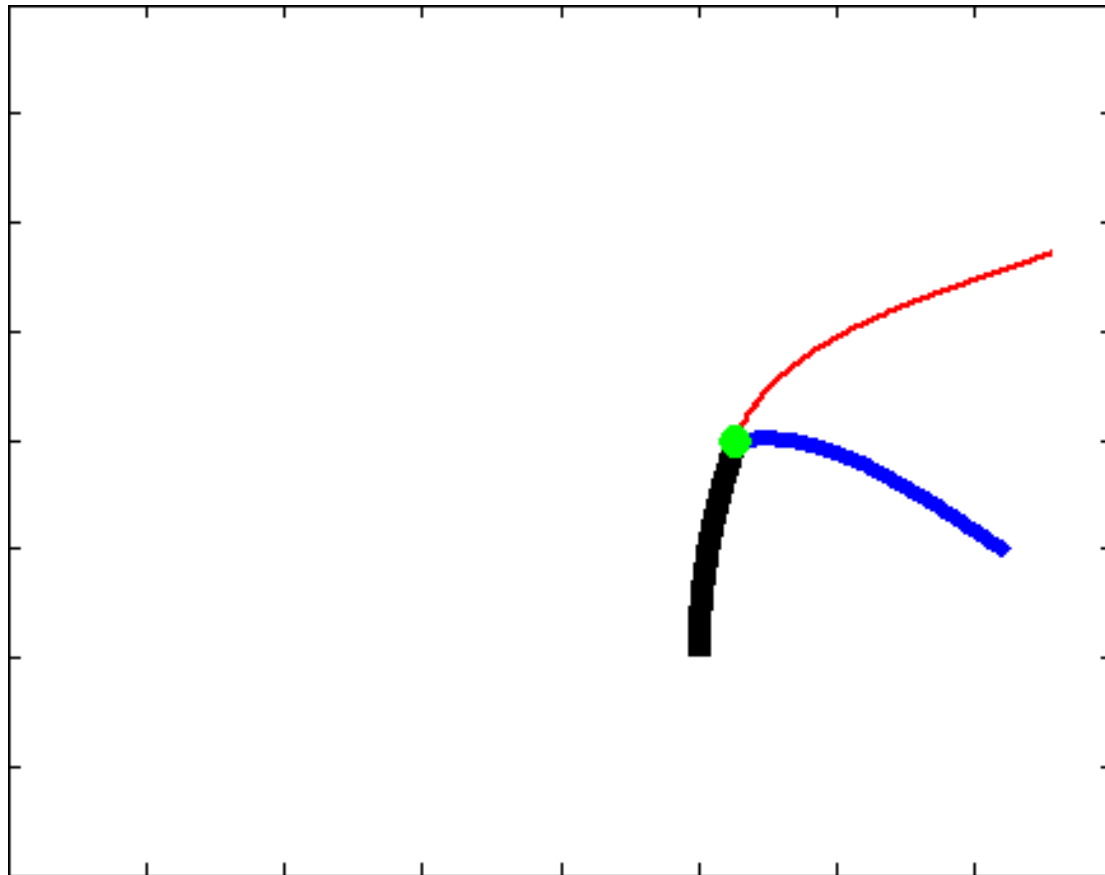


Configurations



# Numerical Branching Method

## DEMO





# 3D S-Curve

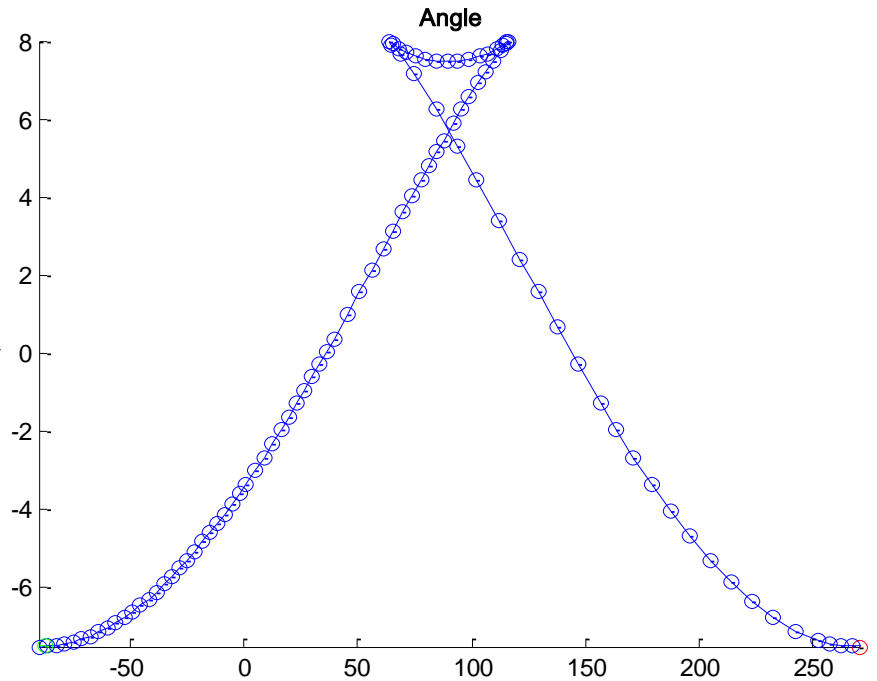
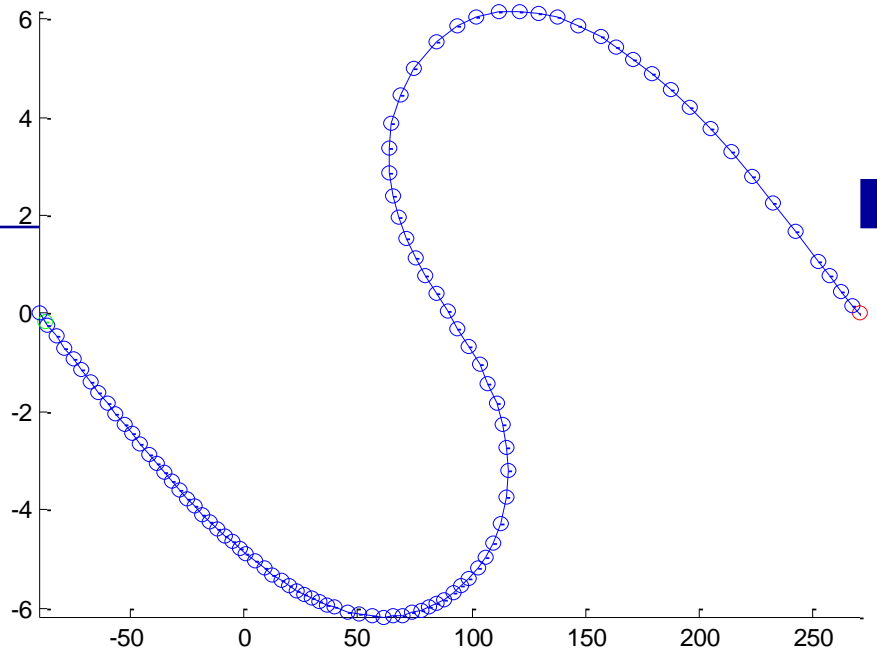
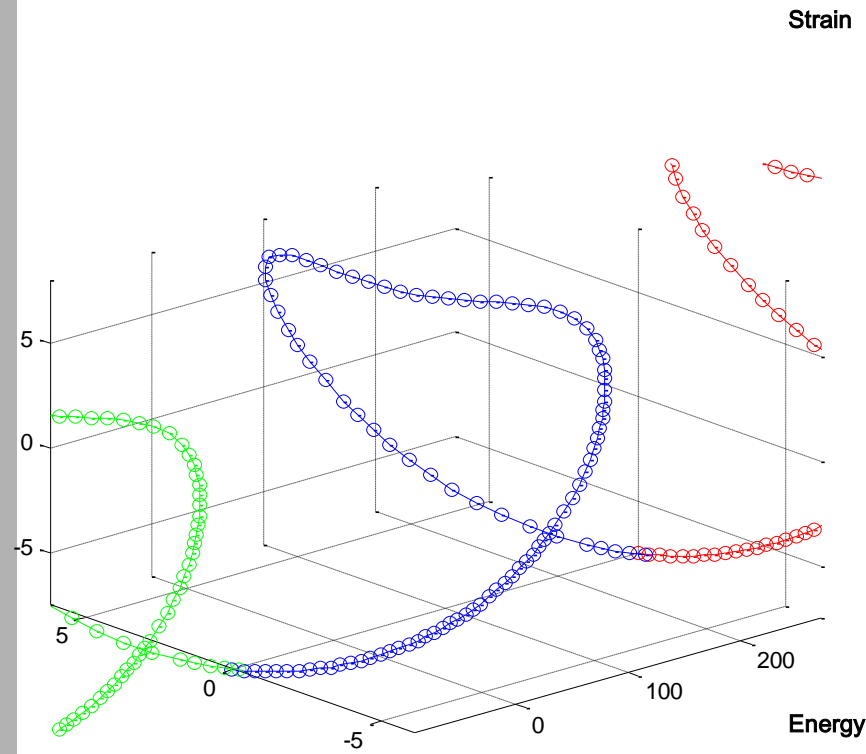
The static Equations of Motion can also be derived with a Variational Method.

$$V = \int_0^L \left[ \frac{1}{2} E(s) I(s) (\theta' - \nu_o(s))^2 + \rho^* g a(s) z(s) + W \sin(\theta) \right] ds$$

$V = \textit{Strain Energy} + \textit{Distributed Potential Energy} + \textit{Singular Potential Energy}$



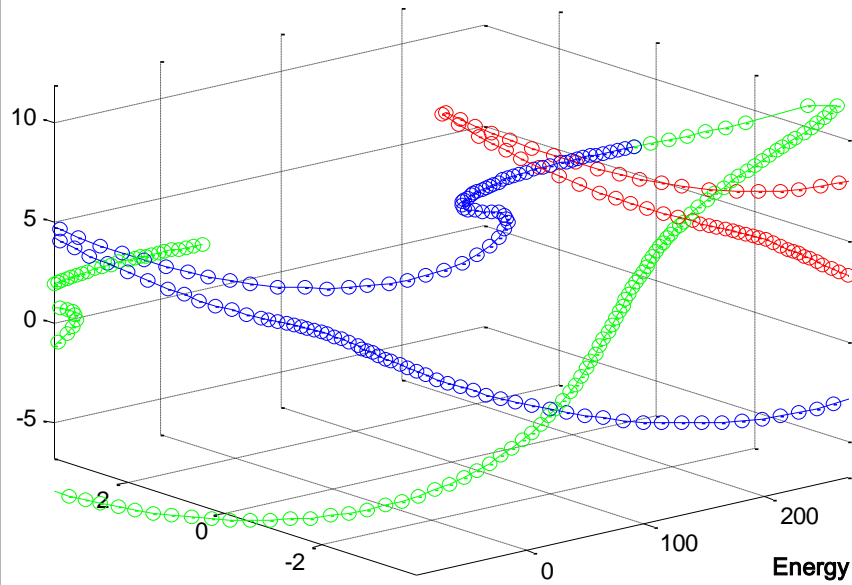
# 3D S-Curve



\*Graphs generated with adaptive step size

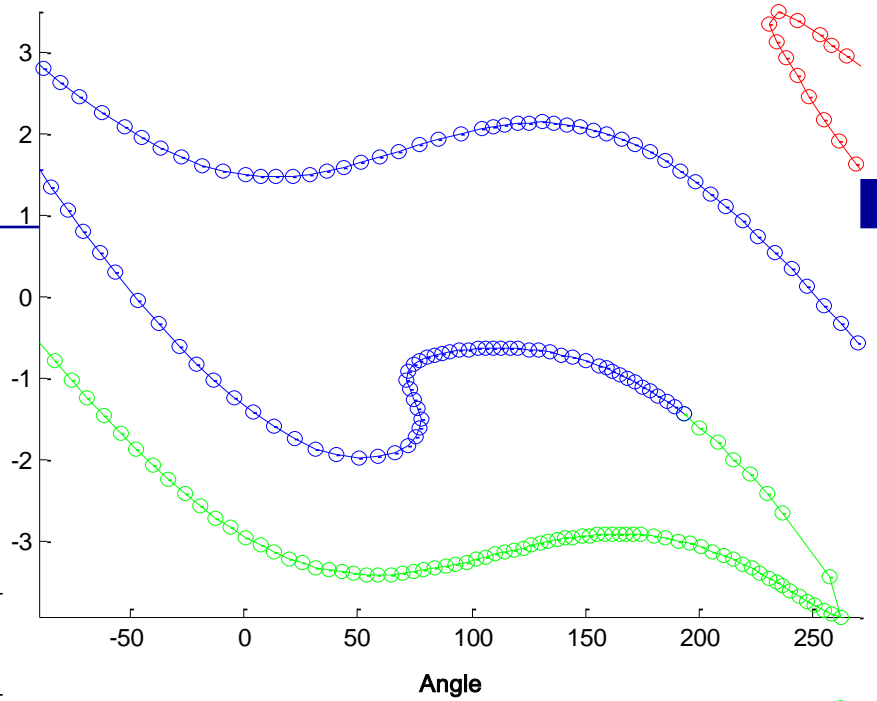


# 3D S-Curve

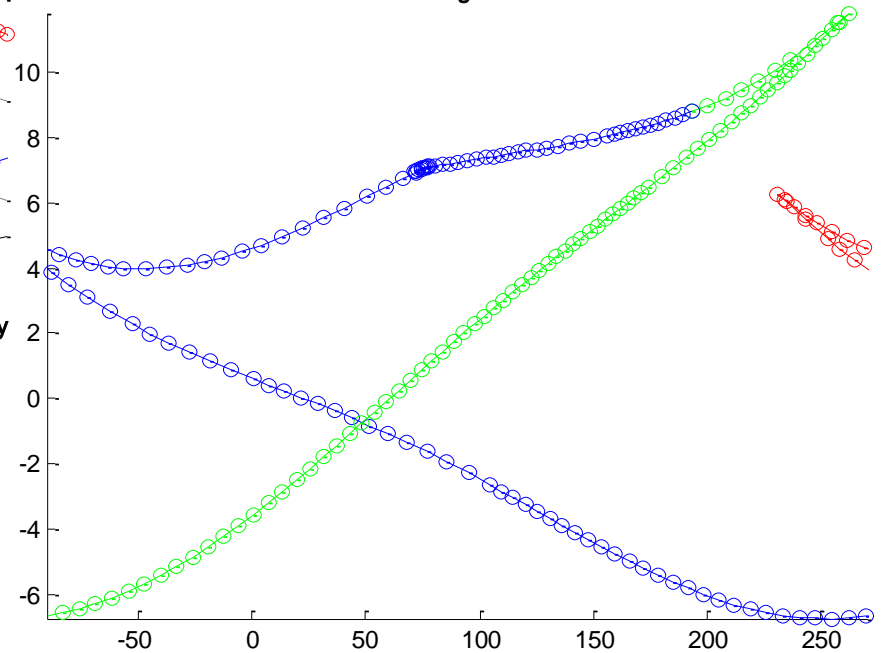


Strain

Energy



Angle





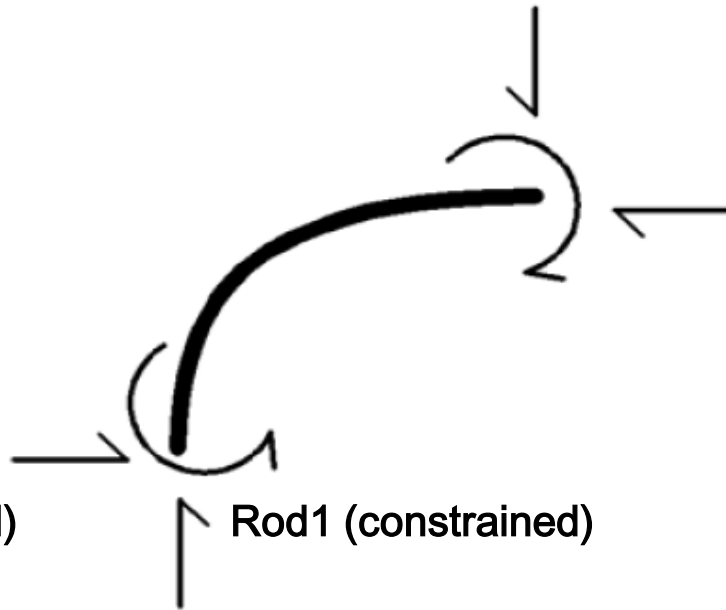
# Growth / Parameter Evolution



# Evolution Analogy



Rod1 (unconstrained)



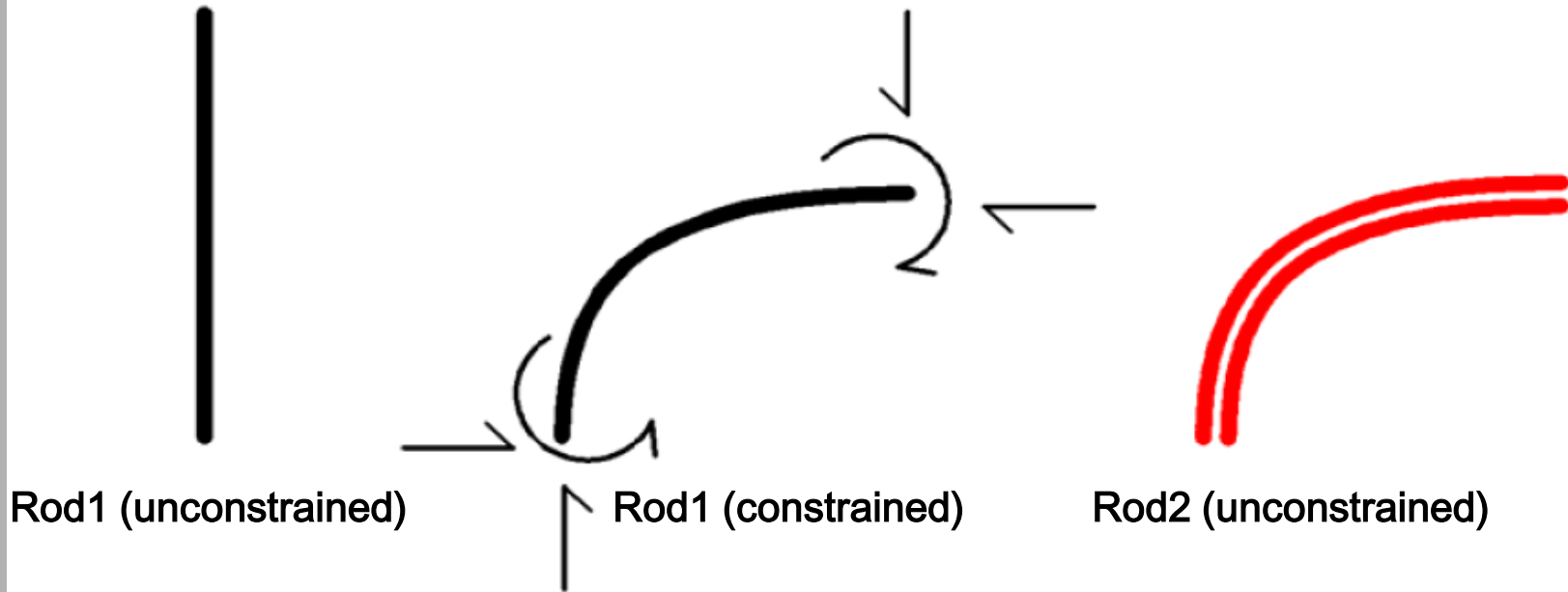
Rod1 (constrained)



Rod2 (unconstrained)



# Evolution Analogy

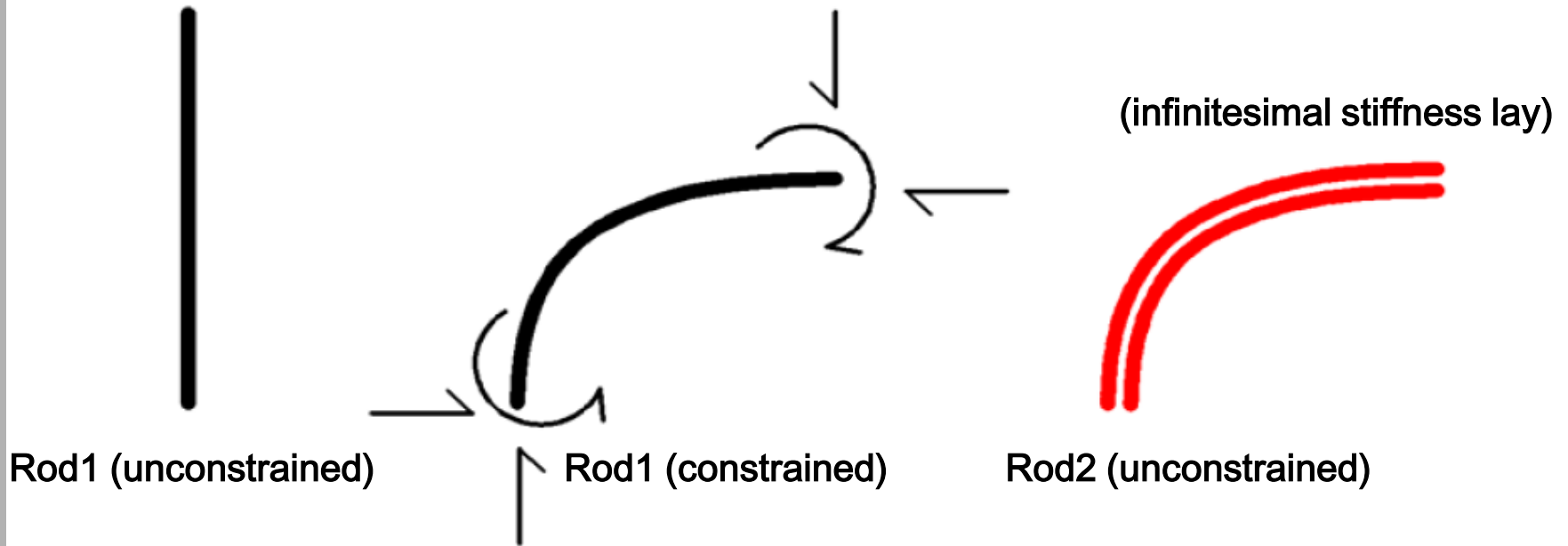


$$\nu_g = \frac{EI_1 \nu_{g1} + EI_2 \nu_{g2}}{EI_1 + EI_2}$$





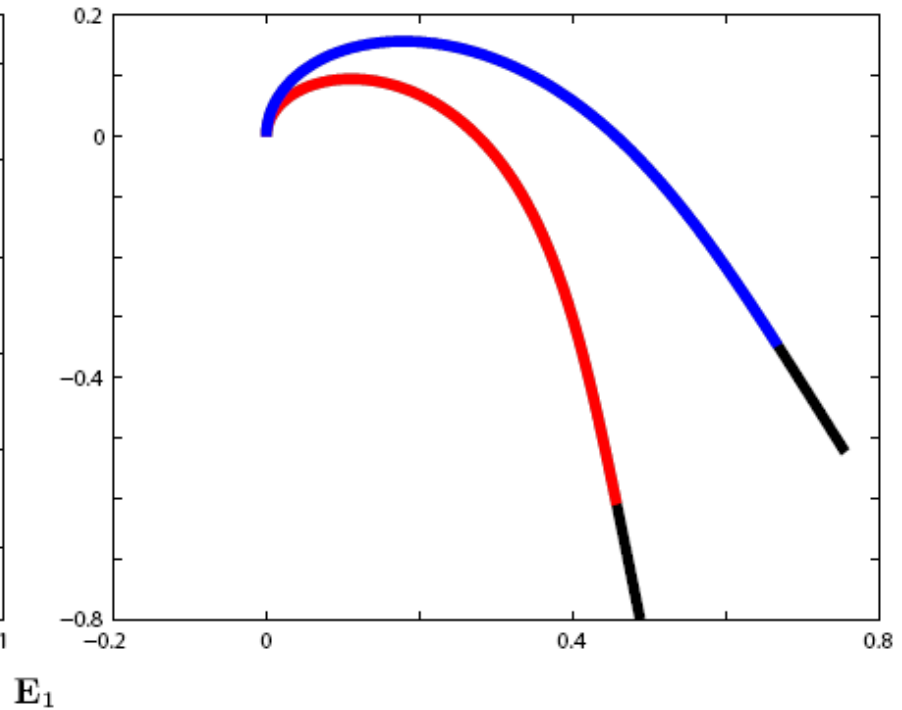
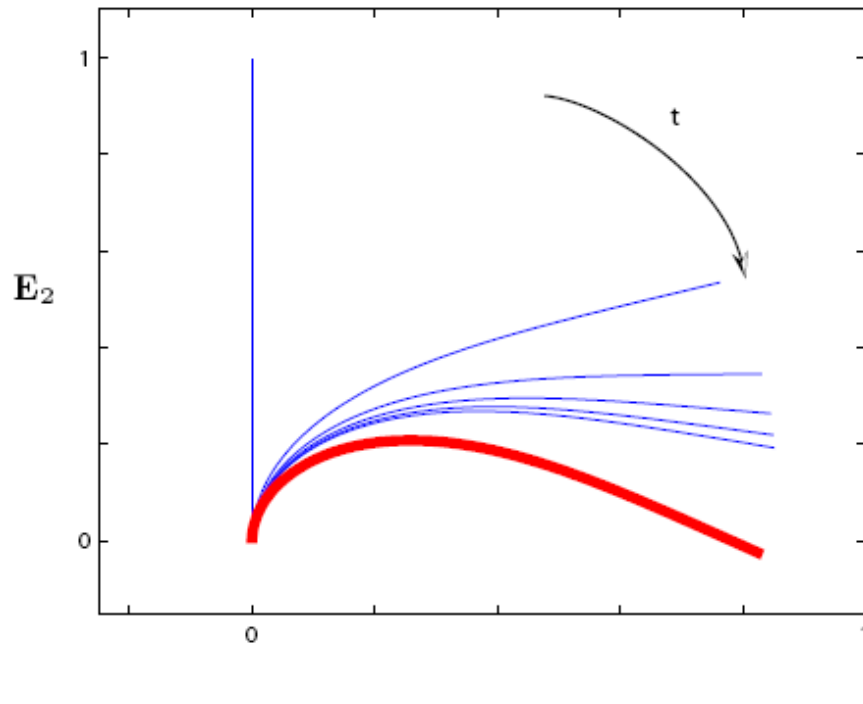
# Evolution Analogy



$$\nu_g(s, t) = \frac{EI_o(s)\nu_g^o(s) + \int_0^t \dot{E}I(s, x)\nu_c(s, x) dx}{EI(s, t)}$$



# Preliminary Numerical Results



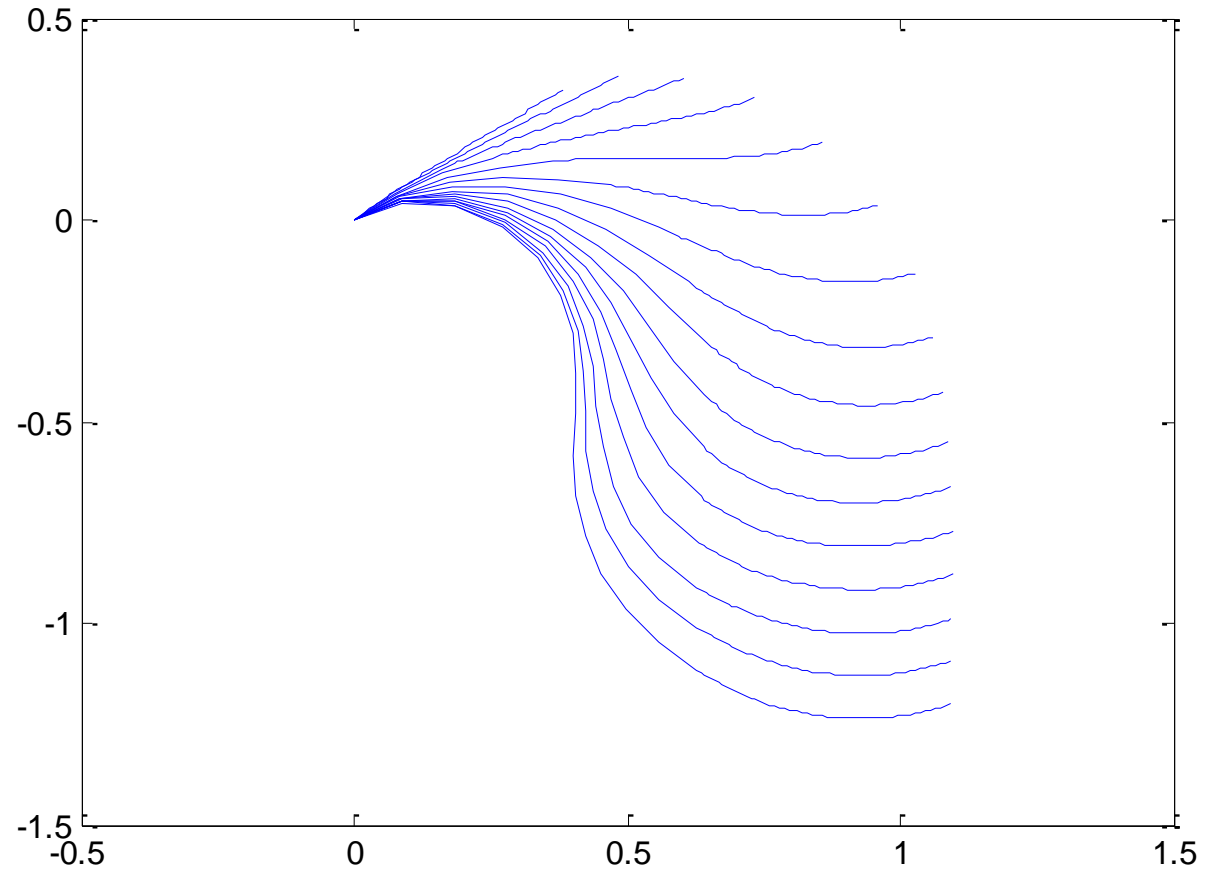
$$V_c = V$$

$$\frac{\partial EI}{\partial t} = \frac{EI(s)_{step} - EI}{\tau}$$



# Gravitopism

Tip Growth with a Preferred Angle



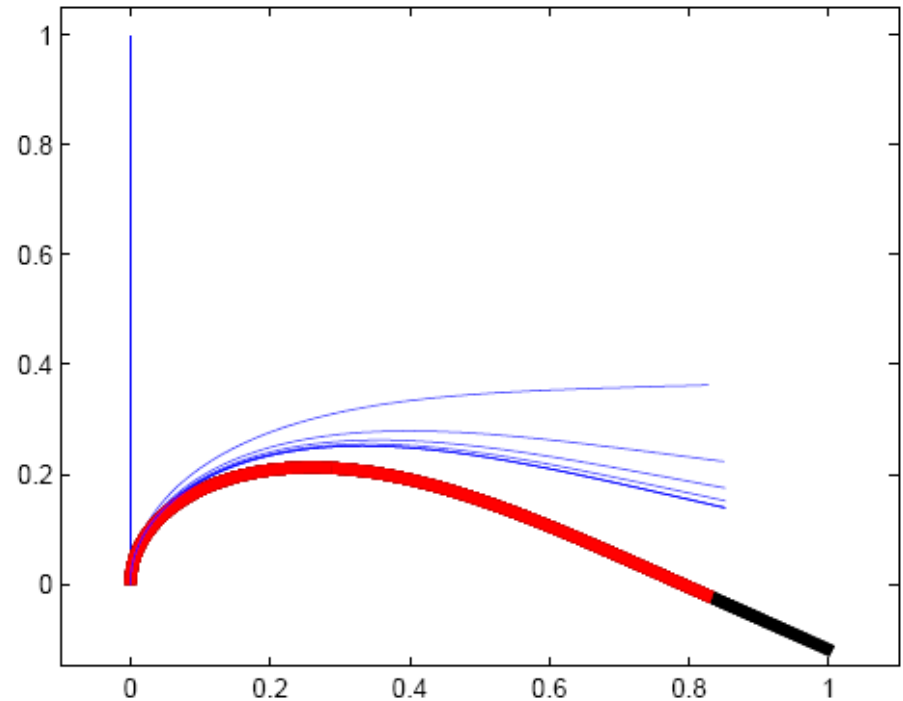


# Prescribe the Output / Find the Input

$$v_c = v_g - \frac{EI}{\dot{EI}} \dot{v}_r$$

Stationary Control Law

$$v_g = v_{EI} + v_{diff}$$



Decomposition of Intrinsic Curvature



# Future Work

- Refining the Control Input to be more realistic
- Combining the Growth and Branching
- Optimizing the Numerical Code