Modeling of Human Response to Ground Motion using Discrete and Continuous Methods

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Final Project
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Motivation For Project

- Using cell phones as sensors for earthquakes
- Effects of human response to rigidly-attached accelerometer signal
- Extraction of ground acceleration from sensor
Discrete Models

ISO standards dictate a vibration simulator of 2DOF systems connected by a rigid frame

Problems with System:

• Not really 2 DOF – just 2 SDOF systems connected by rigid frame

• Suggests that head and torso move independently of the lower body

• Doesn’t account for building motion different from ground motion
Better Discrete Models

Damped 2DOF Model

Problems:

• Hard to estimate parameters
• Doesn’t account for building motion separate from ground motion
• Still a discrete model
Better Discrete Models Continued

Subject-Beam Model

\[ m_s \ddot{X}_s + k_s X_s k_s X_b = 0 \]
\[ -k_s X_s + m_b \ddot{X}_b + (k_s + k_b) X_b = F \]

Problems:

- SDOF – not very robust
- Does not account for damping
Continuous Model

\[ m \frac{\partial^2 U}{\partial t^2} - k \frac{\partial^2 U}{\partial x^2} = 0 \]

- Two rods with independent properties (mass, length, stiffness)
- Assume \( L_1 = L_2 \), \( m_2 = 2m_1 \), and determine stiffnesses
- Force boundary conditions on the system of rods
  - Axial Displacement, Forces =
    - @ 0, no force
    - @ \( x_1 \), no displacement
    - @ \( x_1 + x_2 \),
Solution to PDE System

Separation of Variables

\[ u(x, t) = T(t)\Phi(x) \]

Modal Shape

\[ \Phi_i(x) = C_i \cos b_i x_i + D_i \sin b_i x_i \quad i = 1, 2 \]

Application of BC's

\[ \Phi_1(x_1) = D \sin b_1 x_1 \quad 0 \leq x_1 \leq L_1 \]

\[ \Phi_2(x_2) = D \{\sin b_1 L_1 \cos b_2 x_2 + \sqrt{\frac{m_1 k_i}{m_2 k_2}} \cos b_1 L_1 \sin b_2 x_2\} \quad 0 \leq x_2 \leq L_2 \]
Stiffness Ratio Selection: for simplicity, a ratio of $k_1 / k_2 = 2$ is chosen

Initial Conditions
- No velocity
- Assume sinusoidal initial shape

$$u(x_1, t) = \sum_{i=0}^{n} A_i \sin(b_i x_1) \cos(\omega_i t)$$

$$u(x_2, t) = \sum_{i=0}^{n} A_i \sin(b_i L_1 (1 + \frac{2x_2}{L_2})) \cos(\omega_i t)$$
3 Different Stiffness Ratios

$k_{1}/k_{2}=2$

$k_{1}=k_{2}$

$k_{1}/k_{2}=0.5$
Benefits of Continuous Model

- Reduces the number of parameters to be determined
  - In-between values embedded within the PDE, while discrete requires more nodes (more parameters)
- Can determine relative amplitudes of different modes as a function of location on body.
  - Know most likely position of sensor on human

\[
u(x_1, t) = \sum_{i=0}^{n} A_i \sin(b_i x_1) \cos(\omega_i t)
\]

\[
u(x_2, t) = \sum_{i=0}^{n} A_i \sin(b_i L_1 (1 + \frac{2x_2}{L_2})) \cos(\omega_i t)
\]
Further Study and Conclusions

FURTHER STUDY

- Additional sensors on the human participant to study local vibrational effects.
- Consider lateral movement and horizontal vibrations in order to understand 3-D picture of earthquake motions.
- Refine experiments to determine system parameters.
- Add damping to the model.

CONCLUSIONS

- Discrete models are commonly accepted but do not provide the resolution needed for the project.
- Our continuous model allows for the calculation of the modal response along the length of the body.
- Our continuous model does not meet our original goal of extraction of the earthquake ground motion signal.
http://www.iso.org/iso/catalogue_detail.htm?csnumber=32917


Thanks for listening!
Any questions?