Optimal Highway Traffic Control using a Velocity-Cell Transmission Model

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- Cell Transmission Model
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- Cell Transmission Model for Velocity
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- Flux as a minimization problem
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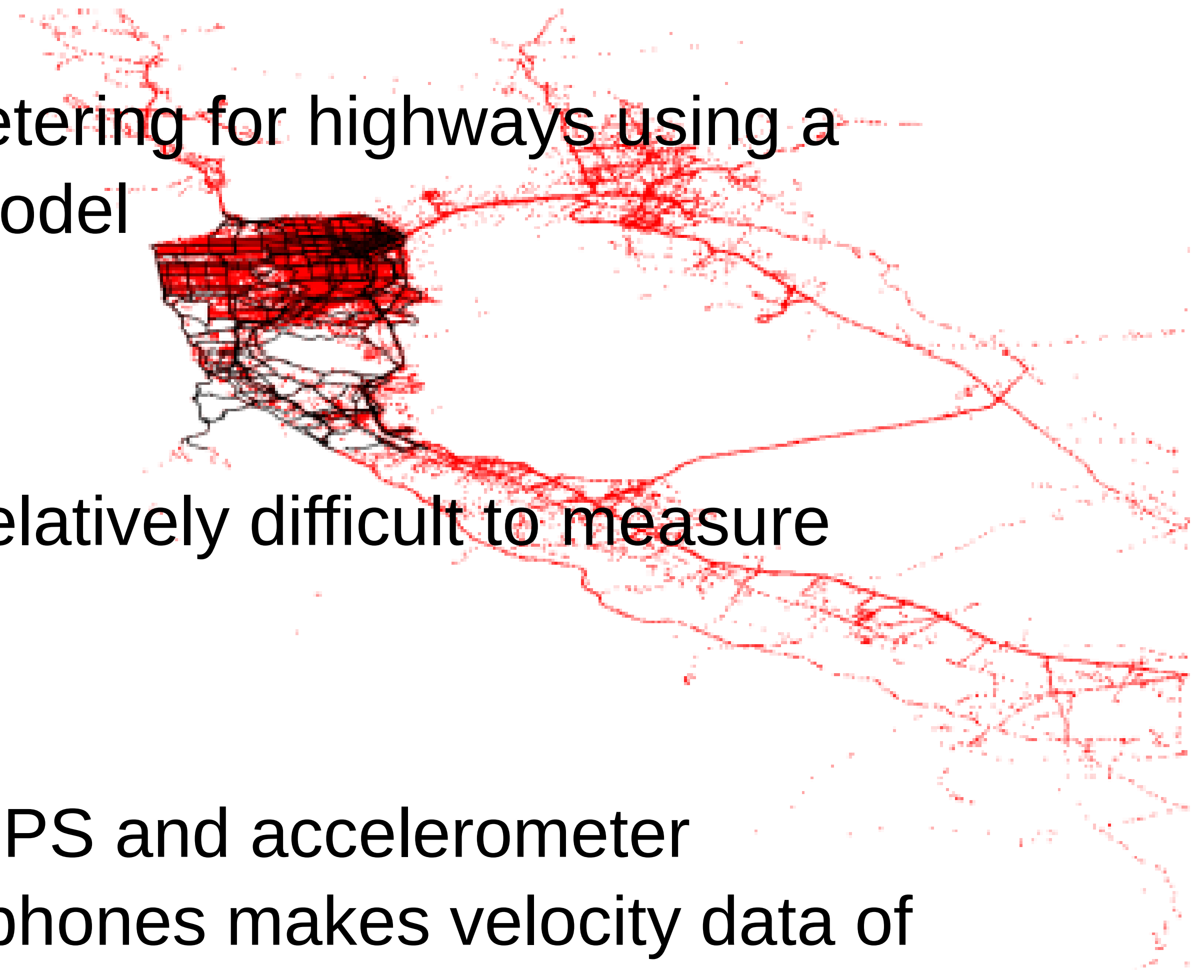
Problem Description and Motivation



- Optimal ramp metering for highways using a velocity based model
- Density data is relatively difficult to measure
- Proliferation of GPS and accelerometer equipped smart phones makes velocity data of vehicles on road available in abundance

Cell Transmission Model

- Discrete approximation to the LWR PDE

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial Q\big(\rho(x,t)\big)}{\partial x} = 0$$

- Highway is divided into discrete cells (indexed by i)

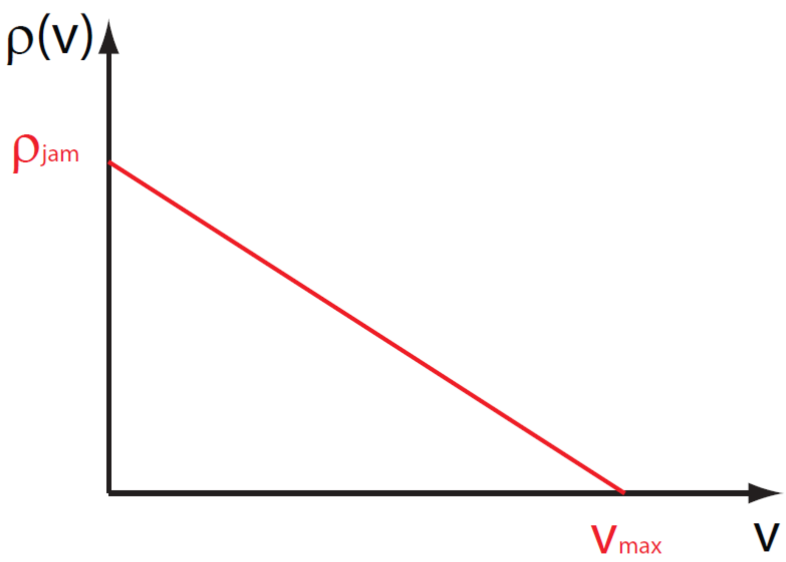
- Density of vehicles in a cell, ρ changes in relation to the flows in and out, G the Godunov flux function

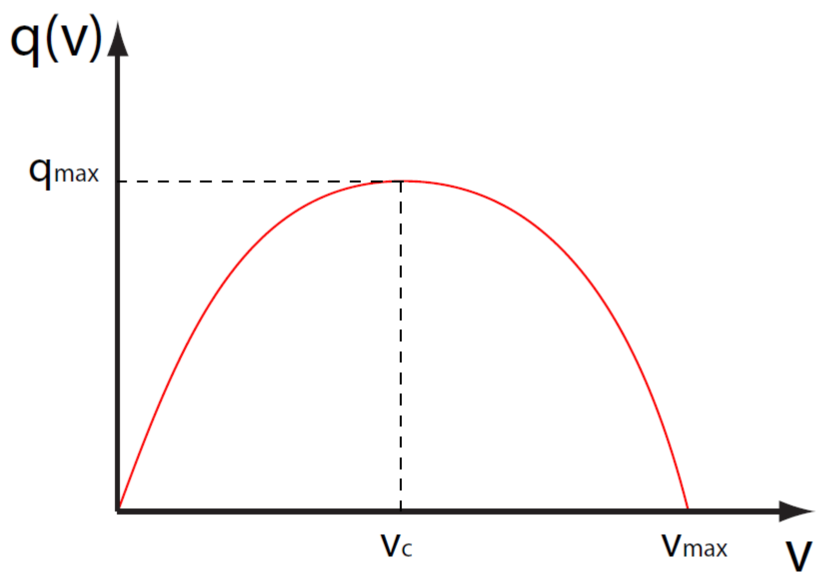
$$\rho_i(k+1) = \rho_i(k) + G_i\,(k) - G_{i+1}(k)$$

Has been shown to be equivalent for piecewise affine flux

Godunov Flux Function

- Our flux function is the Greenshield’s Flux
- Previous work shows that a velocity model will not be equivalent unless the relationship between velocity and density is affine
- Using the relationship $Q = v\rho$ we obtain the following fundamental diagrams

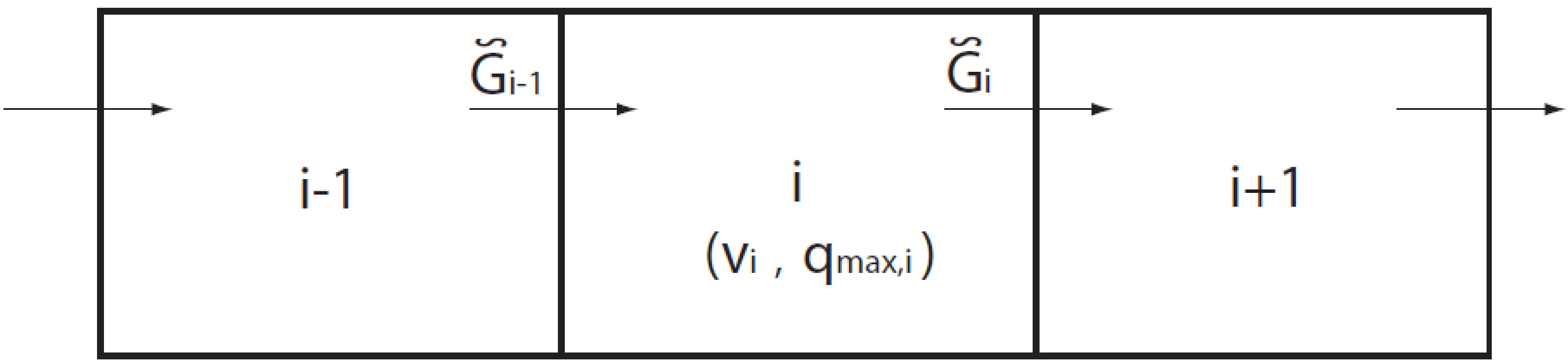




Cell Transmission Model for Velocity

- Combining above equations give the following velocity based model

$$v_i^{n+1} = V\left(V^{-1}(v_i^n) + \frac{\Delta T}{\Delta x}\left[\tilde{G}(v_{i-1}^n, v_i^n) - \tilde{G}(v_i^n, v_{i+1}^n)\right]\right)$$



Godunov Flux

- The Godunov flux is given by:

$$q_i^n = \tilde{G}(v_i^n, v_{i+1}^n) = \begin{cases} \tilde{q}(v_{i+1}), if\ v_c \geq v_{i+1} \geq v_i \\ \tilde{q}(v_c), if\ v_{i+1} \geq v_c \geq v_i \\ \tilde{q}(v_i), if\ v_{i+1} \geq v_i \geq v_c \\ min\{\tilde{q}(v_i), \tilde{q}(v_{i+1})\}, if\ v_i \geq v_{i+1} \end{cases}$$

Optimal Control Problem

Minimize TTT

s.t. $v_i^0, i = 1, \ldots, I$

$$\tilde{G}(v_{-1}, v_0) = \tilde{G}_0$$

$$\tilde{G}(v_I, v_{I+1}) = \tilde{G}_I$$

$$\tilde{G}(v_i, v_{i+1}) = Godunov\ flux$$

- This leads to a problem as our flux is governed by a set of if conditions

Flux as a Minimisation Problem

An equivalent formulation for the flux can be found by:

$$\tilde{G}(v_i, v_{i+1}) = \left| min \left\{ \begin{matrix} sgn(v_i - v_{i+1})q(v_i) \\ sgn(v_i - v_{i+1})q(v_{i+1}) \\ \varepsilon q_{\max} \end{matrix} \right\} \right|$$

$$\varepsilon = \begin{cases} -1, when\ v_i \leq v_c \leq v_{i+1} \\ \geq 1, for\ all\ other\ cases \end{cases}$$

Flux as a Minimisation Problem

- Mixed integer non-linear program

Minimize $J_{\tilde{G}}$

Subject to $J_{\tilde{G}} = \alpha - 2g + \varepsilon_1 + \varepsilon_2$

$$\alpha \geq g$$

$$\alpha \geq -g$$

$$g \leq sgn(v_1 - v_2)q(v_1)$$

$$g \leq sgn(v_1 - v_2)q(v_2)$$

$$g \leq [3 - \varepsilon_1 - \varepsilon_2]q_{max}$$

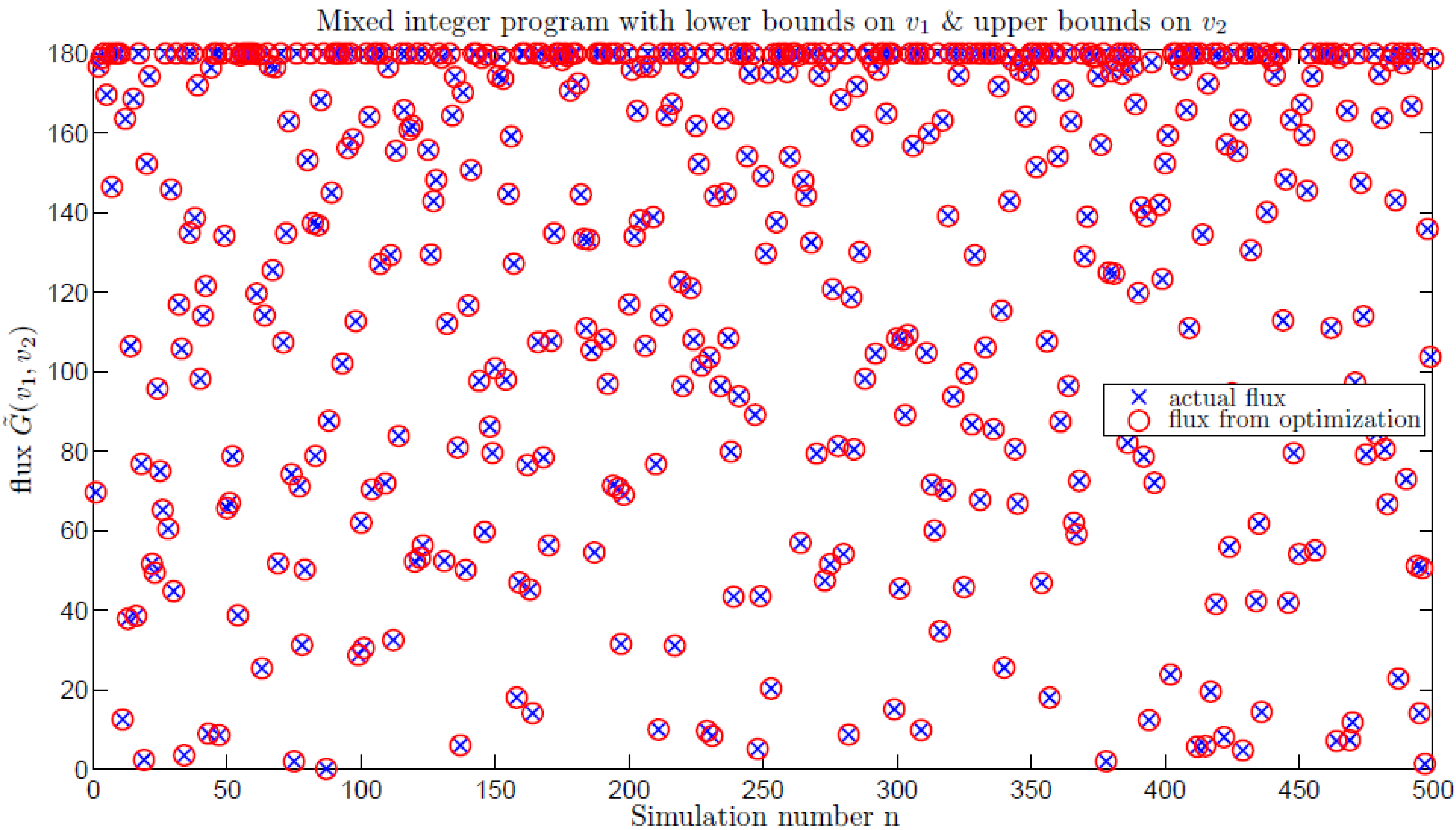
$$\varepsilon_1 \geq 2sgn(v_2 - v_c)$$

$$\varepsilon_1 \geq 0$$

$$\varepsilon_2 \geq 2sgn(v_2 - v_1)$$

$$\varepsilon_2 \geq 0$$

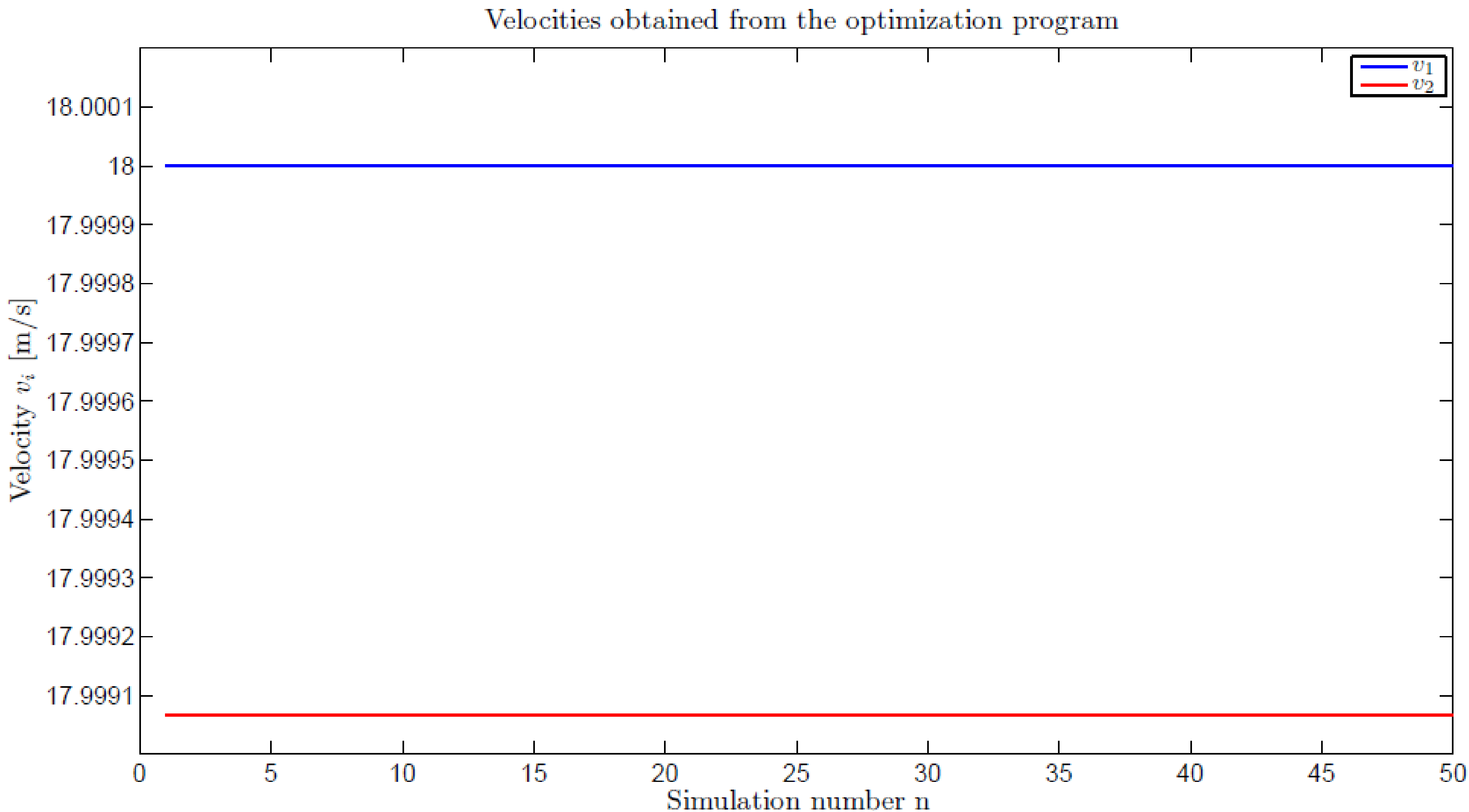
Flux as a Minimisation Problem



Conclusions and Outlook

- We are able to correctly determine flux despite non linearities in the problem
- Slack variables affect our cost
- For control purposes the cost formulation needs to be tweaked or to reformulate the problem such that slack variables are not required (such as Big-M formulation)
- Infeasibilities with solvers when using system dynamics

Additional Material



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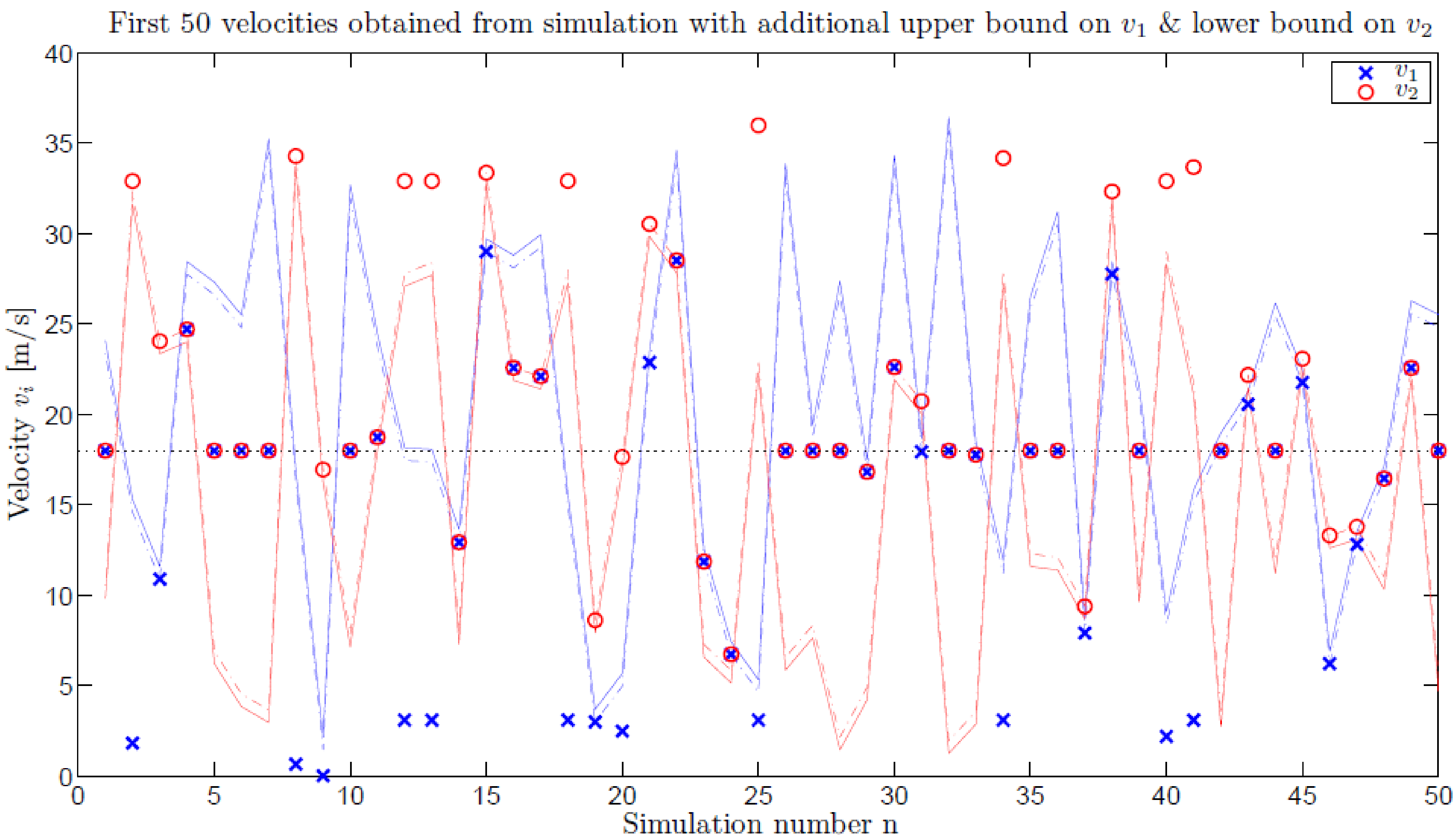

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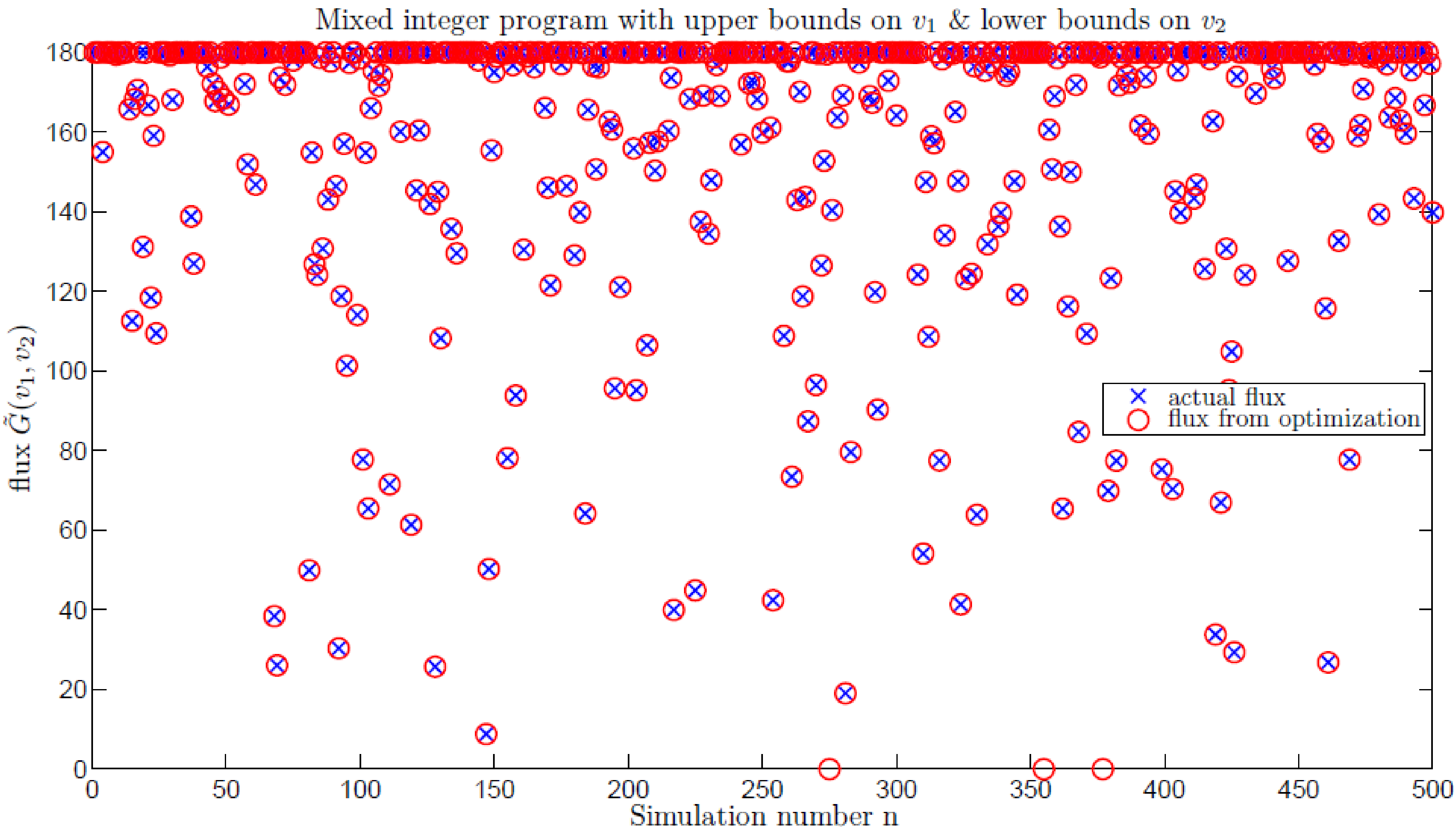
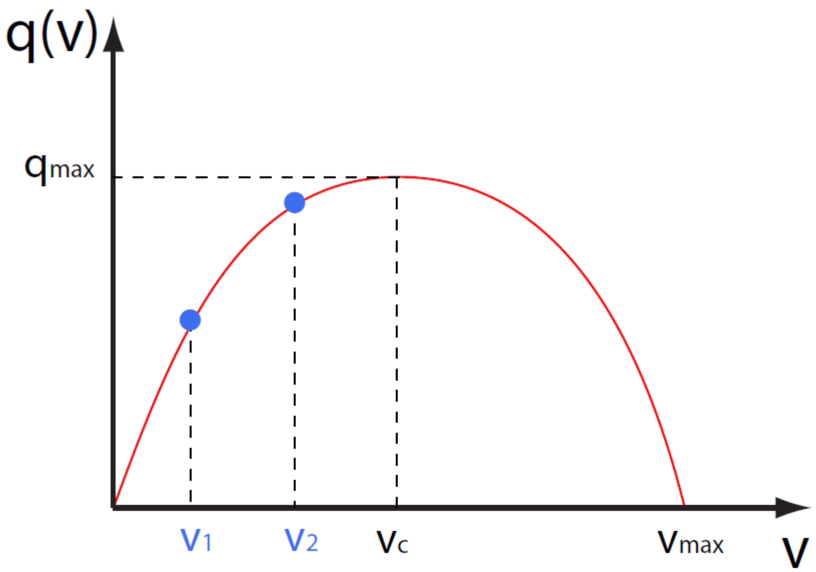


Godunov Flux

$$v_i \leq v_{i+1} \leq v_C \Longrightarrow \widetilde{G}_i = Q(v_2)$$

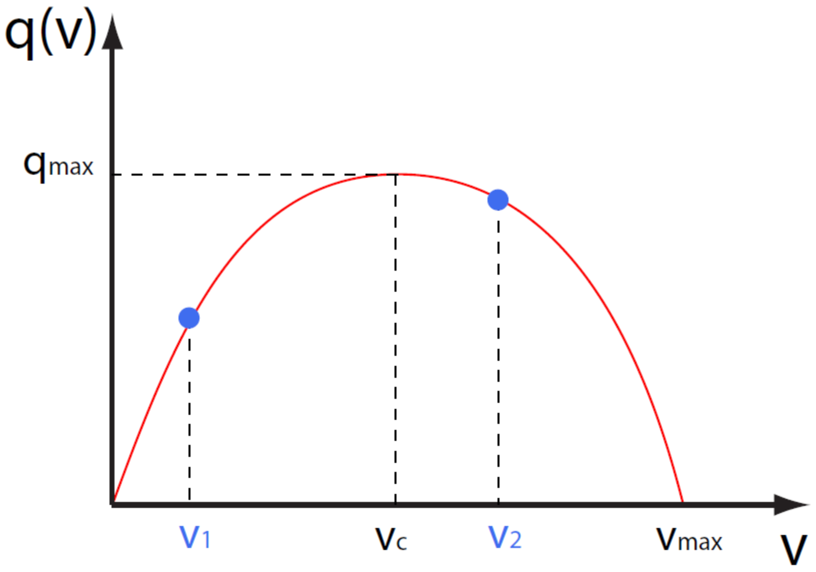
- Current and next cell are both congested
- Cell i+1 is moving faster
- The flux in cell i can be equal to that in cell i+1



Godunov Flux

$$v_i \leq v_c \leq v_{i+1} \Longrightarrow \widetilde{G}_i = Q(v_c)$$

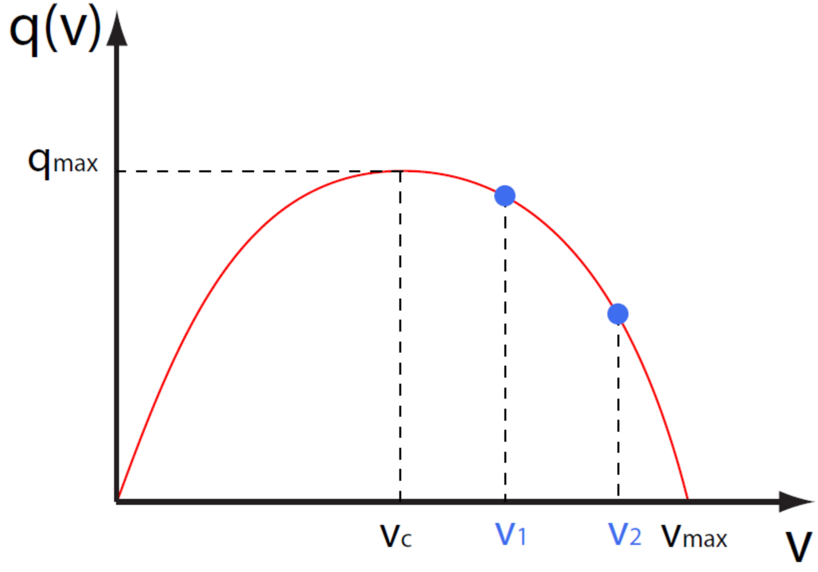
- Cell i is congested
- Cell i+1 is not
- Cell i can discharge at maximum rate



Godunov Flux

$$v_c \leq v_i \leq v_{i+1} \Longrightarrow \widetilde{G}_i = Q(v_1)$$

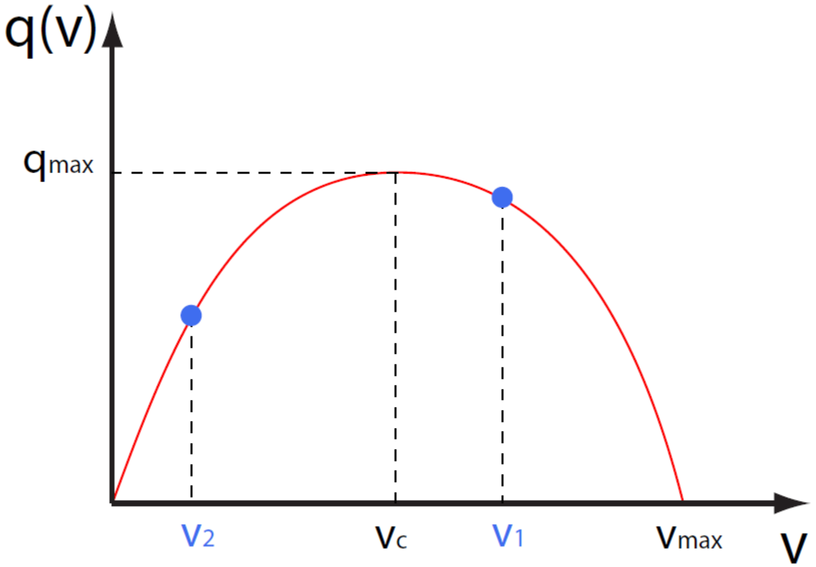
- Both cells are uncongested
- All vehicles can flow from i to i+1



Godunov Flux

$$v_{i+1} \leq v_i \Longrightarrow \widetilde{G}_i = min\{Q(v_1), Q(v_2)\}$$

- Cell i+1 is moving slower than cell I
- The amount that can flow into cell i+1 is upper bounded by the flux of cars outside cell i+1



Cell Transmission Model for Velocity

- Including ramps(control variable)

$$v_i^{n+1} = V\left(V^{-1}(v_i^n) + \frac{\Delta T}{\Delta x}\left[\tilde{G}(v_{i-1}^n, v_i^n) - \tilde{G}(v_i^n, v_{i+1}^n) - R_i^n + S_i^n\right]\right)$$