ME 236 / CE 291F / EE 291
Final Project Presentation

A Precompensation Filter for the Telegraph Equation

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Presentation Outline

- Problem statement
- The physical model
- The transfer function
- Preliminary simulations
- Further work

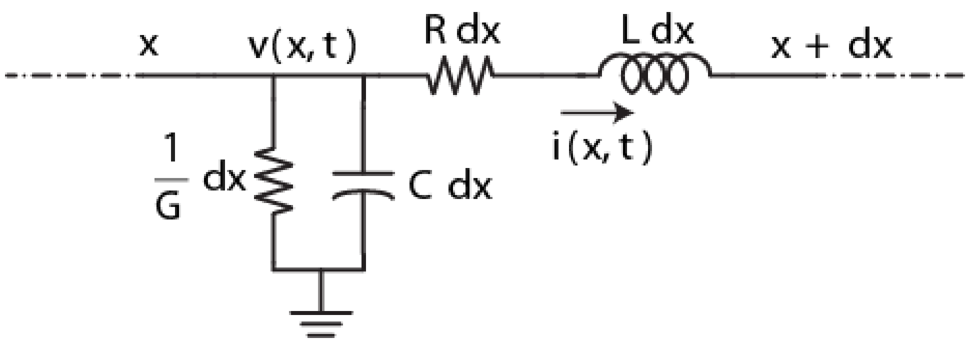
Problem Statement and Motivation

- Consider sending a signal down a transmission line.

- Real lines are not perfect.
  - Resistance, capacitance, inductance, etc.
- As a result, signals become distorted!
- The telegraph equation is a model for describing the parasitic effects along a line.
- Given the model, can we develop a **filter to pre-compensate** for the signal's degradation?

The Physical Model and Telegraph Equation

- We divide the line into infinitesimal pieces.
- Each piece contains the following elements:



- Combining KVL and KCL and eliminating the current yields the telegraph equation:

$$\frac{\partial^2 v(x,t)}{\partial x^2} = LC\frac{\partial^2 v(x,t)}{\partial t^2} + (LG+RC)\frac{\partial v(x,t)}{\partial t} + RGv(x,t)$$

  - BC's: $v(0,t) = u(t) \qquad v(l,t) = Zi(l,t) = y(t)$

The Laplace Transform

- In order to solve the PDE by differential flatness, we apply the Laplace transform:

$$\hat{v}_{xx}(x,s) = LCs^2\hat{v}(x,s) + (LG+RC)s\hat{v}(x,s) + RG\hat{v}(x,s) = \omega(s)\hat{v}(x,s)$$

  - where $\omega(s) = LCs^2 + (LG+RC)s + RG$

- The solution to this 2nd-order ODE is well known:

$$\hat{v}(x,s) = A(s)\cosh\left(\sqrt{\omega(s)}x\right) + B(s)\sinh\left(\sqrt{\omega(s)}x\right)$$

- We take the "Laplaced" B.C.s to solve for constants:

$$\hat{v}(0,s) = \hat{u}(s) \qquad \hat{v}(l,s) = Z\hat{i}(l,s) = -\frac{Z}{R+sL}\hat{v}_x(l,s) = \hat{y}(s)$$

- Substitution and algebra yield

$$\hat{u}(s) = \left(\cosh(\sqrt{\omega(s)}l) + \frac{R+sL}{Z}\frac{\sinh(\sqrt{\omega(s)}l)}{\sqrt{\omega(s)}}\right)\hat{y}(s)$$

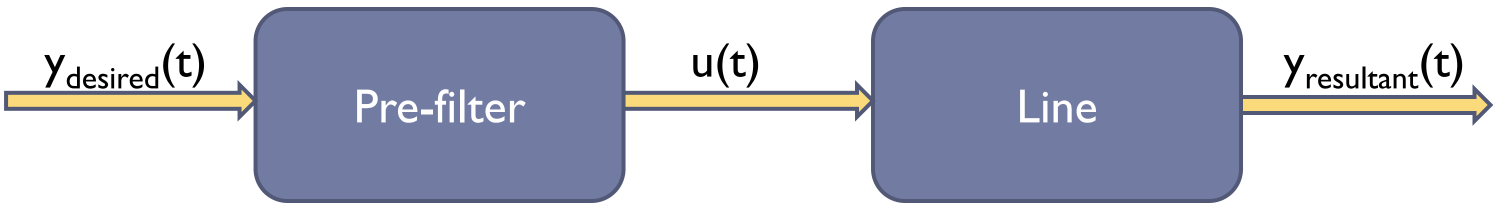
Time Domain Solution

- Assume $G = 0$; let $\lambda = l\sqrt{LC}$ and $\alpha = R/2L$
- Then we can use the inverse Laplace transforms and various identities to obtain

$$u(t) = \frac{1}{2}e^{-\alpha\lambda}\left(1 - \frac{1}{Z}\sqrt{\frac{L}{C}}\right)y(t-\lambda) + \frac{1}{2}e^{\alpha\lambda}\left(1 + \frac{1}{Z}\sqrt{\frac{L}{C}}\right)y(t+\lambda)$$
$$+ \int_{-\lambda}^{\lambda}\{\frac{R}{4Z\sqrt{LC}}e^{-\alpha\tau}J_0(i\alpha\sqrt{\tau^2-\lambda^2})$$
$$+ \frac{e^{-\alpha\tau}i\alpha}{2\sqrt{\tau^2-\lambda^2}}\left(\lambda - \frac{1}{Z}\sqrt{\frac{L}{C}}\tau\right)J_1(i\alpha\sqrt{\tau^2-\lambda^2})\}y(t-\tau)d\tau$$

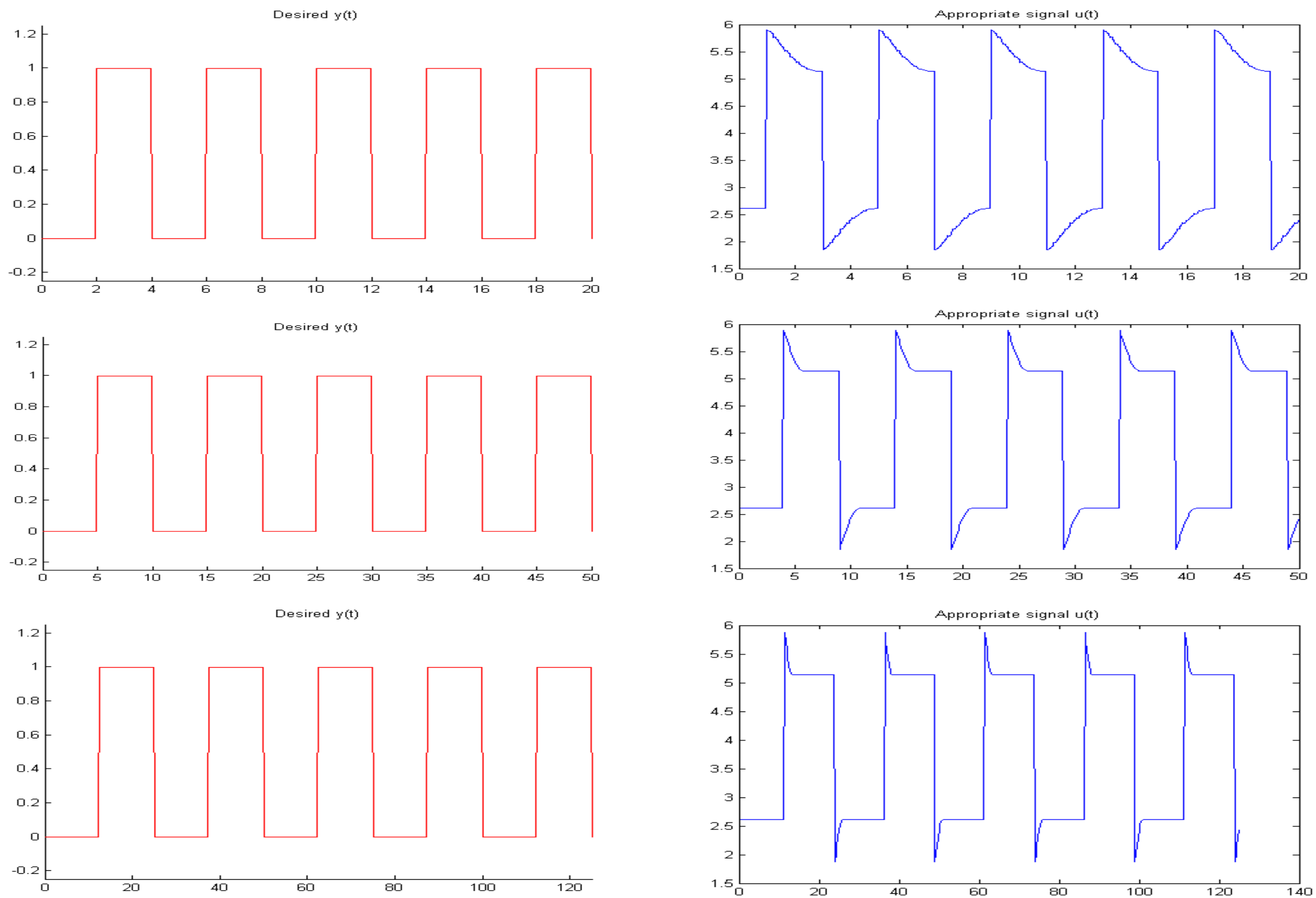
Simulation Paradigm

- We seek to prove the effectiveness of the pre-filter by simulation.



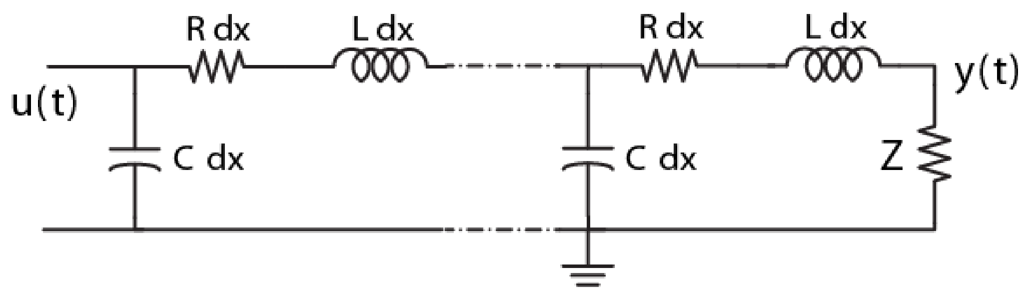
- Entails two simulations: the pre-filter and the line
  - Prefilter simulation: Computation of the time-domain pre-filter equation (MATLAB)
  - Line simulation: Discretization of the telegraph equation into its "infinitesimal" components and solving the circuit (SPICE)
- Success if $y_{desired}(t) = y_{resultant}(t)$

Pre-filter Simulation Results



Transmission Line Simulation Work

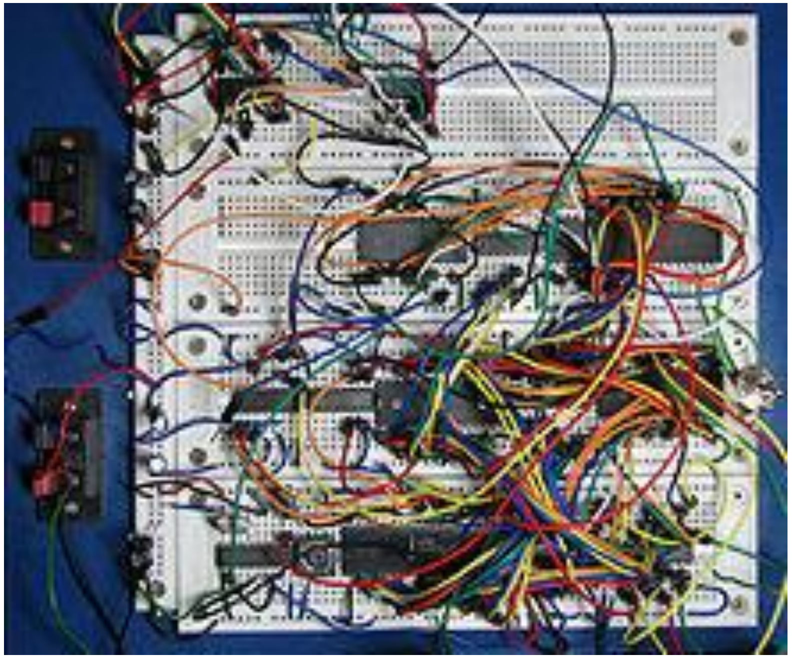
- The circuit is set up with N discrete elements (see below).
- The final element is capped with a load impedance Z.



- The input voltage to the system will be the u(t) signal outputted by the pre-filter simulation.
- We have simulated with COMSOL, SPICE, and two homebrewed algorithms, but more work is necessary before presentation of final results.

Physical Circuit

- We would like to create a real circuit of what is modeled in SPICE and test it with the same inputs.
  - Need to find appropriately valued components



References

- M. Fliess, P. Martin, N. Petit, P. Rouchon, "Active signal restoration for the telegraph equation", in Proc. of the 38th IEEE Conf. on Decision and Control, 1999.
- J. Feldman, "Derivation of the Telegraph Equation", 2005.
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