# IMAGE SEGMENTATION USING CHAN-VESE LEVEL SET METHOD AND COMPARISON WITH OTHER METHODS

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# PROBLEM DEFINITION

- Present and Explain the Chan-Vese algorithm PDE based level set method
- Describe a method to implement it.
- Compare the Chan-Vese algorithm with:
  - Thresholding
  - Edge based detection
  - K-means clustering

# NOTATIONS AND DEFINITIONS

Variational level set method:

- Level set function: $\phi(i, j, t)$

  $$\phi(i, j) = (x - 50)^2 + (y - 50)^2 - 600$$

- Functional: $F : \phi \longrightarrow \mathbb{R}$

  $$F[\phi] = \int_\Omega H(\phi)\, I$$

# PDES AND PROBLEM SOLUTION METHOD

Chan-Vese Functional to be minimized

$$F(c_1, c_2, \phi) = \mu \int_\Omega \delta(\phi(x,y)) |\nabla\phi(x,y)|\, dx\, dy \; + \nu \int_\Omega H(\phi(x,y))\, dx\, dy$$
$$+ \lambda_1 \int_\Omega |u_0(x,y) - c_1|^2 H(\phi(x,y))\, dx\, dy \; + \lambda_2 \int_\Omega |u_0(x,y) - c_2|^2 (1 - H(\phi(x,y)))\, dx\, dy.$$

$$c_1 = \frac{\int_\Omega I \cdot H(\phi)\, dx\, dy}{\int_\Omega H(\phi)\, dx\, dy} \qquad c_2 = \frac{\int_\Omega I \cdot (1 - H(\phi))\, dx\, dy}{\int_\Omega (1 - H(\phi))\, dx\, dy}$$

# PDES AND PROBLEM SOLUTION METHOD

Euler Lagrange formulation

$$\partial_\phi \mathcal{F} = -\delta_\varepsilon(\phi)\left[\mu\nabla\cdot\frac{\nabla\phi}{|\nabla\phi|} - \nu - \lambda_1(u_0-c_1)^2 + \lambda_2(u_0-c_2)^2\right] = 0$$

Final PDE after Performing gradient decent

$$\frac{\partial\phi}{\partial t} = -\partial_\phi \mathcal{E} = \delta_\varepsilon(\phi)\left[\mu\nabla\cdot\frac{\nabla\phi}{|\nabla\phi|} - \nu - \lambda_1(u_0-c_1)^2 + \lambda_2(u_0-c_2)^2\right]$$

**BC:** $\frac{\delta_\varepsilon(\phi)}{|\nabla\phi|}\frac{\partial\phi}{\partial\vec{n}} = 0 \text{ on } \partial\Omega$

**IC:** $\phi(0, x, y) = \phi_0(x, y) \text{ in } \Omega$

# IMPLEMENTING THE PDE

## Discretising variables

$$H_\varepsilon(z) = \frac{1}{2}\left(1 + \frac{2}{\pi}\arctan\left(\frac{z}{\varepsilon}\right)\right) \qquad \delta_\varepsilon(z) = H'_\varepsilon(z) \qquad \phi_t = (\phi^{n+1}_{i,j} - \phi^n_{i,j})/\Delta t$$

$$\Delta^x_+ \phi^n_{i,j} = \phi^n_{i+1,j} - \phi^n_{i,j} \qquad \Delta^x_- \phi^n_{i,j} = \phi^n_{i,j} - \phi^n_{i-1,j}$$

$$\Delta^y_+ \phi^n_{i,j} = \phi^n_{i,j+1} - \phi^n_{i,j} \qquad \Delta^y_- \phi^n_{i,j} = \phi^n_{i,j} - \phi^n_{i,j-1}$$

Dicretised PDE:

$$\frac{\phi^{n+1}_{i,j} - \phi^n_{i,j}}{\Delta t} = \delta_h(\phi^n_{i,j})\frac{\mu}{h^2}(p \cdot L(\phi^n)^{p-1})\left[\Delta^x_-\left(\frac{\Delta^x_+\phi^{n+1}_{i,j}}{\sqrt{(\Delta^x_+\phi^n_{i,j})^2/h^2 + (\phi^n_{i,j+1} - \phi^n_{i,j-1})^2/(2h)^2}}\right)\right.$$

$$\left. + \Delta^y_-\left(\frac{\Delta^y_+\phi^{n+1}_{i,j}}{\sqrt{(\phi^n_{i+1,j} - \phi^n_{i-1,j})^2/(2h)^2 + (\Delta^y_+\phi^n_{i,j})^2/h^2}}\right)\right]$$

$$- \delta_h(\phi^n_{i,j})\left(\nu + \lambda_1(I_{i,j} - c_1(\phi^n))^2 - \lambda_2(I_{i,j} - c_2(\phi^n))^2\right).$$

# IMPLEMENTING THE PDE

After Rearranging PDE (many steps)

$$\phi_{i,j}^{n+1} = F_1 \phi_{i+1,j}^{n+1} + F_2 \phi_{i-1,j}^{n+1} + F_3 \phi_{i,j+1}^{n+1} + F_4 \phi_{i,j-1}^{n+1} + F p_{i,j}$$

Can be solved using an iterative method.

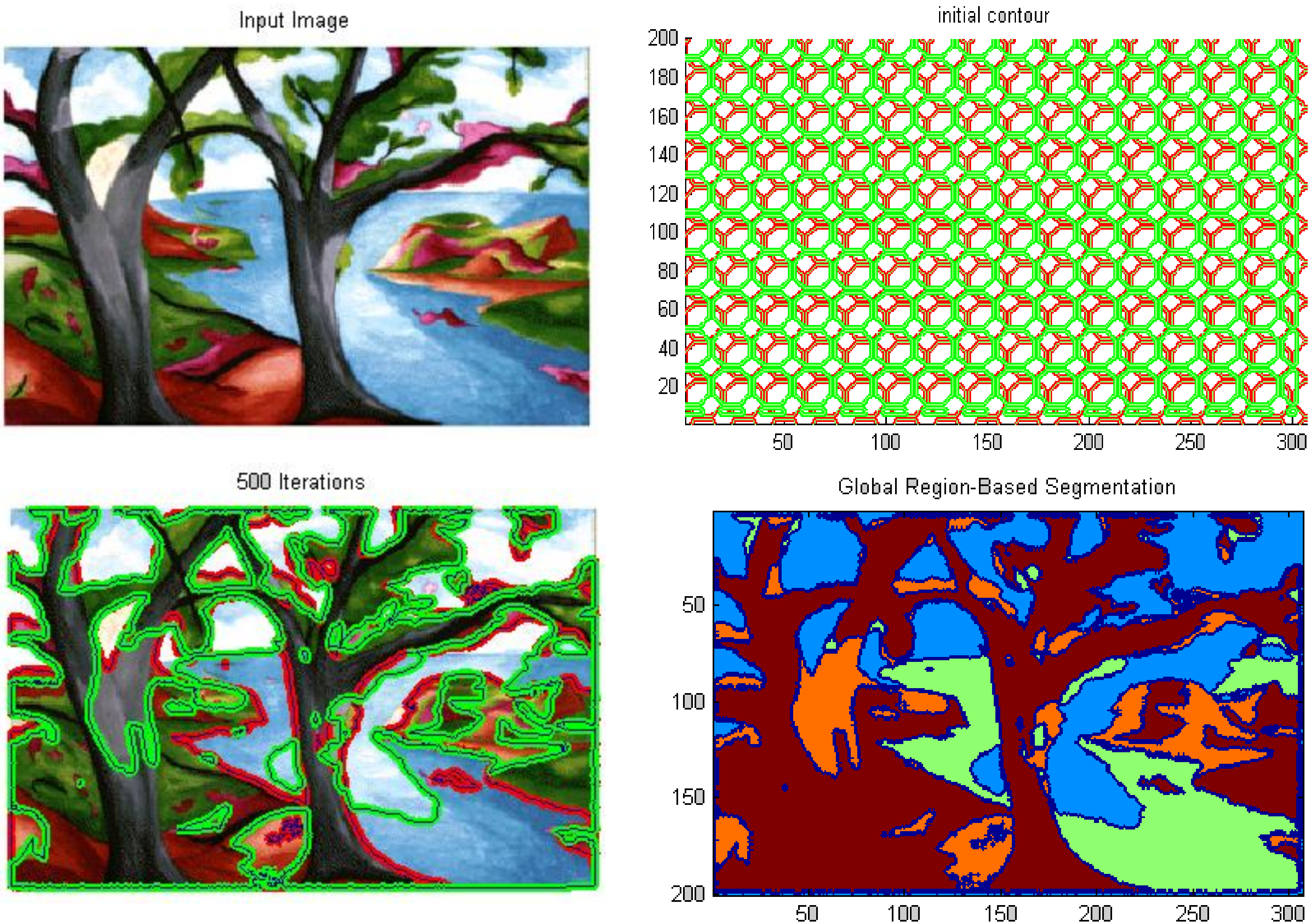
Final algorithm important steps:

Set $\phi_0 = \phi 0$

Solve the PDE i $\phi^n$ to obtain $\phi^{n+1}$

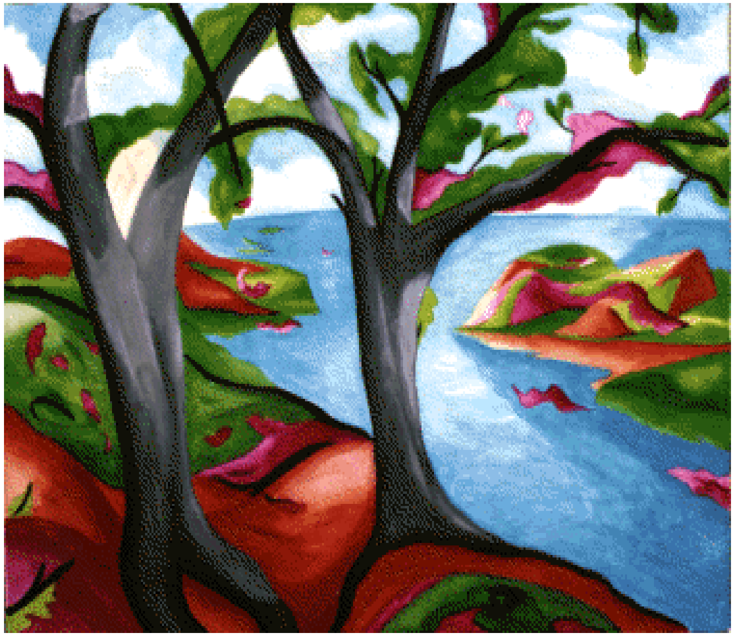
Check if solution is stationary. If not n=n+1 and repeat.

# COMPARING CHAN-VESE TO OTHER SEGMENTATION ALGORITHMS



# COMPARING CHAN-VESE TO OTHER SEGMENTATION ALGORITHMS

- Thesholding: define a threshold brightness

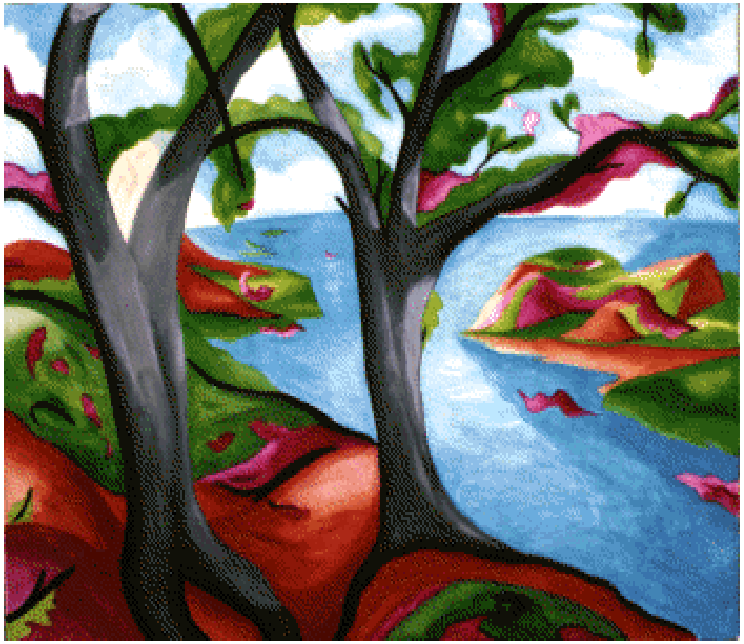


Input image

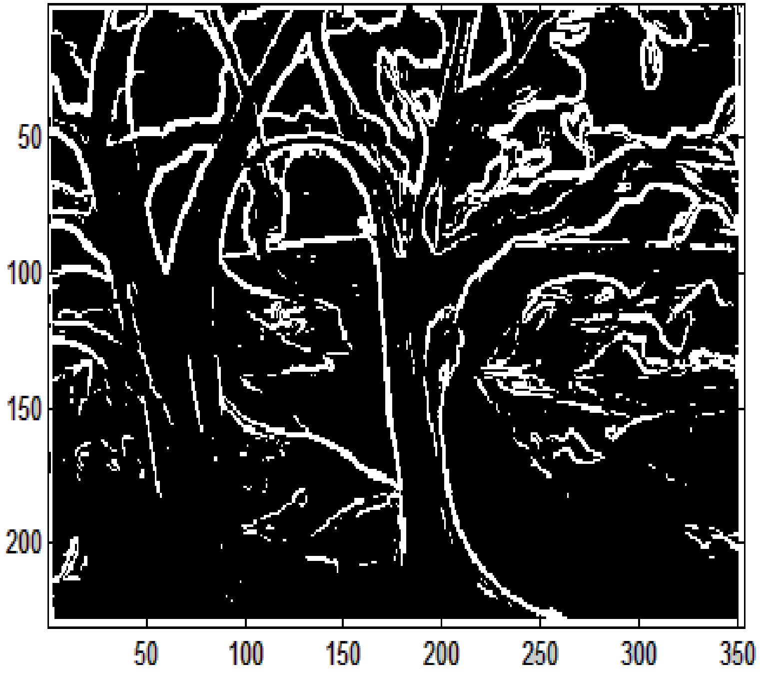
Threshold segmentation

# COMPARING CHAN-VESE TO OTHER SEGMENTATION ALGORITHMS

• Edge based detection: discontinuities in gray-level, color and texture



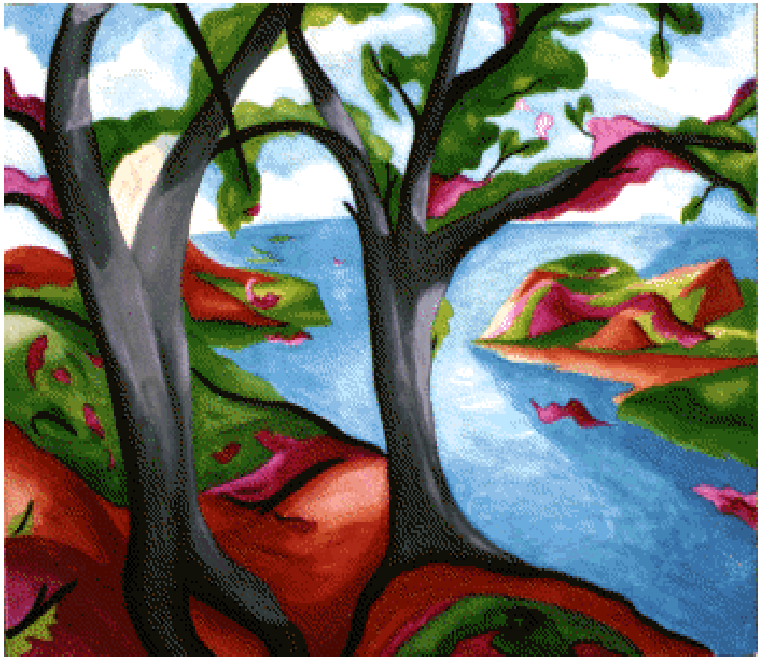
Input image




Sovel edge based detection

# COMPARING CHAN-VESE TO OTHER SEGMENTATION ALGORITHMS

- K-means clustering: partition the image in k groups



Input image

4-means segmentation

# FUTURE STEPS AND CONCLUSIONS

- Use specific metrics for comparing algorithms:
- Normalized Probabilistic Rand (NPR) index
- Stability with respect to changes in parameter settings and with respect to different images
- Test on Berkeley image segmentation database
(300 images with their ground truth)

# THANK YOU

- Questions?