Application of piezoelectric tiles in traffic energy harvesting

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Derivation of Lighthill-Whitham-Richards (LWR) PDE

$$\frac{\partial \rho(x,t)}{\partial t} + q'(\rho(x,t))\frac{\partial \rho(x,t)}{\partial x} = 0, \qquad (1)$$

- ρ = vehicle density on the highway
- q(ρ) = flux function

<u>Greenshield flux function</u>

$$q(\rho) = v\rho\left(1 - \frac{\rho}{\rho *}\right) \qquad (2)$$

- ρ* = jam density
- v = free flow density

(CONTINUE LWR EQUATION)

- Differentiate the Greenshield flux equation and substitute back to the LWR equation
- We obtain:

$$\frac{\partial \rho(x,t)}{\partial t} + v\left(1 - \frac{2\rho(x,t)}{\rho*}\right)\frac{\partial \rho(x,t)}{\partial x} = 0. \tag{3}$$

METHOD OF CHARACTERISTICS

- Introduce two new variables

$$\xi = \xi(x, t) \tag{4}$$

$$\eta = \eta(x, t) \tag{5}$$

- Differentiate ρ with respect to x and t thus yields the following

$$\frac{\partial \rho(x,t)}{\partial t} = \frac{\partial \rho(x,t)}{\partial \xi(x,t)}\frac{\partial \xi(x,t)}{\partial t} + \frac{\partial \rho(x,t)}{\partial \eta(x,t)}\frac{\partial \eta(x,t)}{\partial t} \tag{6}$$

$$\frac{\partial \rho(x,t)}{\partial x} = \frac{\partial \rho(x,t)}{\partial \xi(x,t)}\frac{\partial \xi(x,t)}{\partial x} + \frac{\partial \rho(x,t)}{\partial \eta(x,t)}\frac{\partial \eta(x,t)}{\partial x} \tag{7}$$

(CONTINUE METHOD OF CHARACTERISTICS)

- Plug (6), (7) back to

$$\frac{\partial \rho(x,t)}{\partial t} + v\left(1 - \frac{2\rho(x,t)}{\rho*}\right)\frac{\partial \rho(x,t)}{\partial x} = 0. \tag{3}$$

- We get:

$$\frac{\partial \rho(x,t)}{\partial \xi(x,t)}\frac{\partial \xi(x,t)}{\partial t} + \frac{\partial \rho(x,t)}{\partial \eta(x,t)}\frac{\partial \eta(x,t)}{\partial t} + v\left(1 - \frac{2\rho(x,t)}{\rho*}\right)\left(\frac{\partial \rho(x,t)}{\partial \xi(x,t)}\frac{\partial \xi(x,t)}{\partial x} + \frac{\partial \rho(x,t)}{\partial \eta(x,t)}\frac{\partial \eta(x,t)}{\partial x}\right) = 0 \tag{8}$$

(CONTINUE METHOD OF CHARACTERISTICS)

- Regroup the terms to obtain the following form:

$$\left(\frac{\partial\xi(x,t)}{\partial t}+v\left(1-\frac{2\rho(x,t)}{\rho*}\right)\left(\frac{\partial\xi(x,t)}{\partial x}\right)\right)\frac{\partial\rho(x,t)}{\partial\xi(x,t)}+$$
$$\left(\frac{\partial\eta(x,t)}{\partial t}+v\left(1-\frac{2\rho(x,t)}{\rho*}\right)\left(\frac{\partial\eta(x,t)}{\partial x}\right)\right)\frac{\partial\rho(x,t)}{\partial\eta(x,t)}=0 \qquad (9)$$

(CONTINUE METHOD OF CHARACTERISTICS)

- We seek curves of constant η, so we set the total differential of η to be zero:

$$d\eta = \frac{\partial \eta(x,t)}{\partial x} dx + \frac{\partial \eta(x,t)}{\partial t} dt = 0 \tag{10}$$

- In order to get rid of the solution dependency on η, we want

$$\frac{\partial \eta(x,t)}{\partial t} + v\left(1 - \frac{2\rho(x,t)}{\rho*}\right)\frac{\partial \eta(x,t)}{\partial x} = 0 \tag{11}$$

- Now we have a set of two linear homogeneous equations.

(CONTINUE METHOD OF CHARACTERISTICS)

- A nontrivial solution is obtained by setting the following determinant to zero: :

$$\begin{vmatrix} dt & dx \\ 1 & v\left(1 - \frac{2\rho(x,t)}{\rho*}\right) \end{vmatrix} = 0 \qquad (12)$$

- Solving (12), we get:

$$dx - v\left(1 - \frac{2\rho(x,t)}{\rho*}\right) dt = 0 \qquad (13)$$

- which can be re-written as:

$$\frac{dx}{dt} = v\left(1 - \frac{2\rho(x,t)}{\rho*}\right) \qquad (14)$$

(CONTINUE METHOD OF CHARACTERISTICS)

- Since ρ is constant along the characteristics, we know that $\rho(x,t) = \rho_0(x_0)$ if the characteristic curve at time t = 0 goes from x_0 to (x,t).
- Integrating (14) with respect to time yields the following:

$$x - x_0 = v\left(1 - \frac{2\rho_0(x_0)}{\rho*}\right)t \tag{15}$$

- which can be written as:

$$x_0 = x - vt + \frac{2v\rho_0(x_0)}{\rho*}t \tag{16}$$

(CONTINUE METHOD OF CHARACTERISTICS)

- Since ρ is constant along the characteristics, we know that $\rho(x,t) = \rho_0(x_0)$ if the characteristic curve at time $t = 0$ goes from x_0 to (x,t).
- Integrating (14) with respect to time yields the following:

$$x - x_0 = v\left(1 - \frac{2\rho_0(x_0)}{\rho*}\right)t \qquad (15)$$

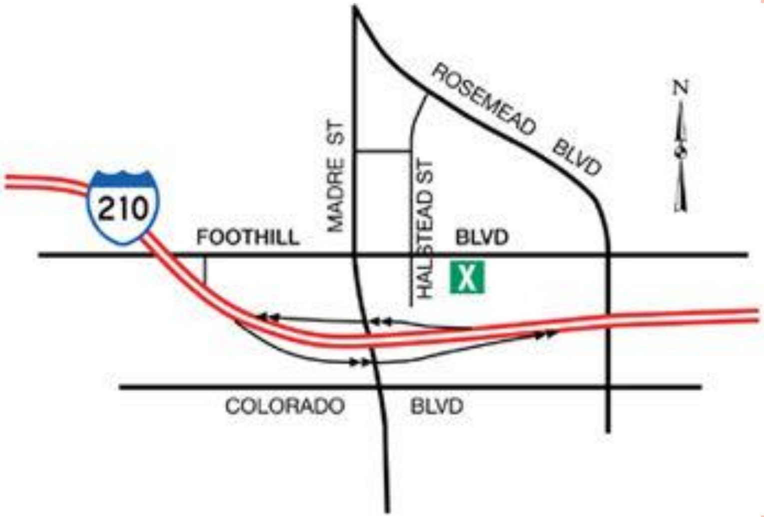
- or:

$$x_0 = x - vt + \frac{2v\rho_0(x_0)}{\rho*}t \qquad (16)$$

The equation for x_0 is now given in terms of x and t, which is easy to solve given simple ρ_0 expressions

TOPL AND CTMSIM

- **T**ools for **O**perational **Pl**anning
- CTMSIM is an interactive freeway traffic macro-simulator for MATLAB developed by the TOPL Group at UC Berkeley.
- The model is based on the Asymmetric Cell Transmission Model (ACTM).
- Collects data on I-210 freeway



ACTM

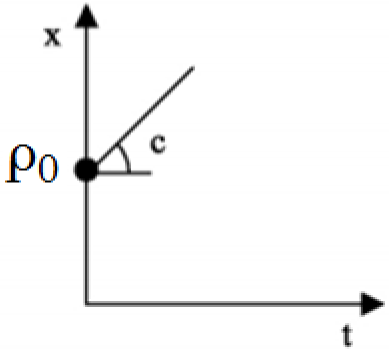
- To understand how CTMSIM works, we first take a look at the ACTM.
- First, the freeway is divided into cells, each having at most one on-ramp and/or one off-ramp junction.
  - On-ramp junction allows cars to enter the cell
  - Off-ramp junction allows cars to leave the cell
  - In a cell with both on-ramp and off-ramp junctions, the on-ramp junction must always be upstream of the off-ramp junction.

CTMSIM

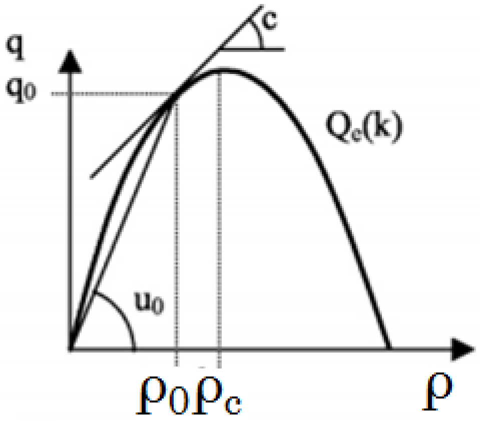
- In CTMSIM, we are given the following data for each cell:
  - Post mile at cell start
  - Post mile at cell end
  - Cell capacity
  - Critical density
  - Jam density
  - On-ramp flow
  - On-ramp capacity
  - Off-ramp split ratio
  - Off-ramp capacity
  - Initial density of all cells
  - Inflow of vehicles at the first cell and all the first cell and on-ramp junctions
  - Outflow of vehicles at all off-ramp junctions

3 DIFFERENT TRAFFIC STATES

- Free flow ($\rho_0 < \rho < \rho_c$)
  - u = speed of the traffic stream
  - c = characteristic, or solution line
  - $u_0 > u > u_c$
  - $c > 0$
    - Characteristics run in the same direction as the traffic flow.
    - The properties of the traffic flow propagate in the same direction as the traffic flow



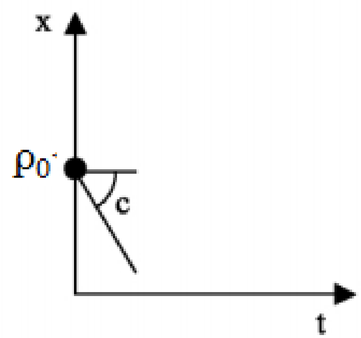
the t-x diagram



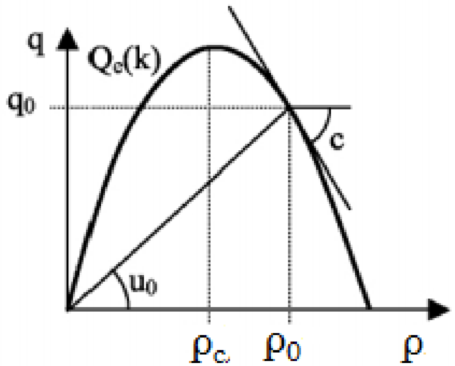
the ρ-q fundamental diagram

(CONTINUE) 3 DIFFERENT TRAFFIC STATES

- Congested flow ($\rho_c < \rho < \rho_j$)
  - u = speed of the traffic stream
  - c = characteristic, or solution line
  - $u_j < u < u_c$ (u_j = speed at maximum density ρ_j)
  - $c < 0$
    - Characteristics run in the opposite direction as the traffic flow.
    - The properties of the traffic flow propagate against direction as the traffic flow



the t-x diagram



the ρ-q fundamental diagram

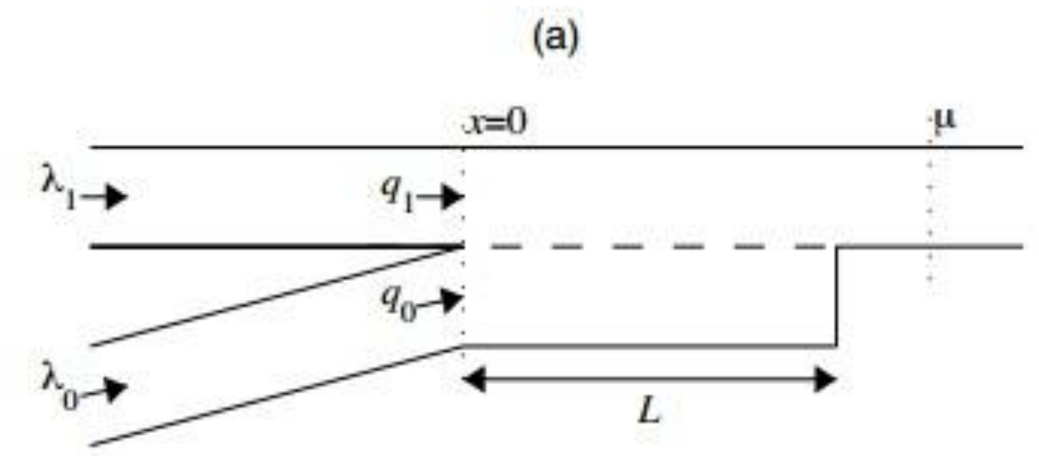
(CONTINUE) 3 DIFFERENT TRAFFIC STATES

- Capacity flow ($\rho = \rho_c$)
  - maximal flow rate
  - $c = 0$
    - This regime cannot propagate in either direction relative to the traffic stream.
    - Capacity flow remains at the same location and functions as an upstream boundary for congested flow and a downstream boundary for free flow.

Problem of using Greenshields flux function

- Fails to determine the flows exiting two branch roadways and merging to flow through a single roadway

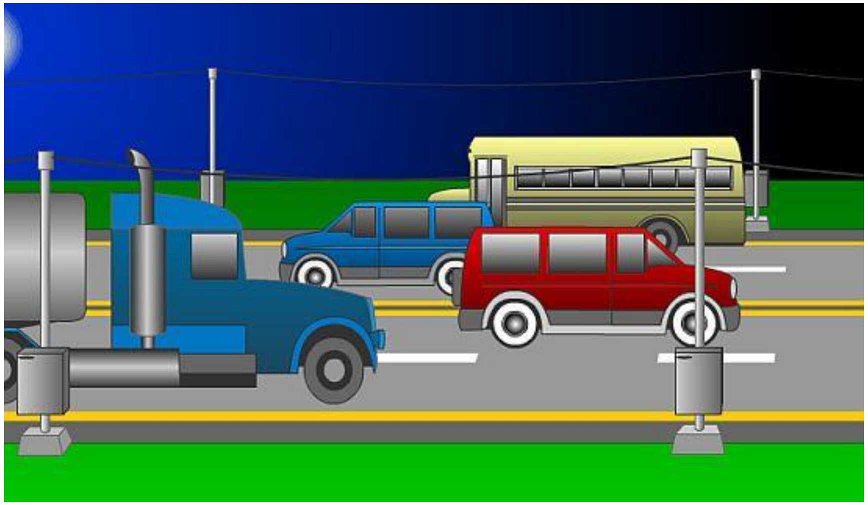
- Newell-Daganzo Merge Model



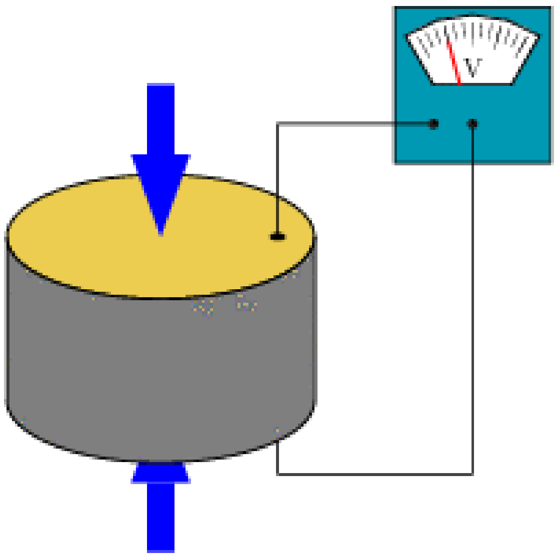
Applying TOPL on Energy Harvesting System

- The Typical California Highway:
  - Average Daily Traffic: 11,520 to 51,773 vehicles per day

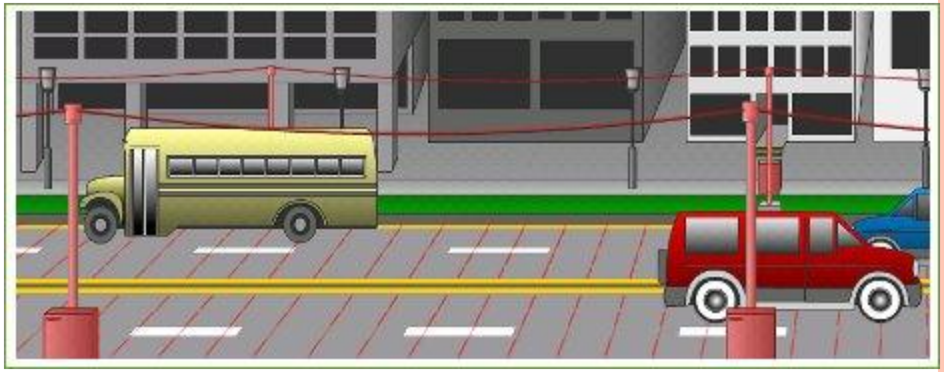
- The Idea:
  - Is it possible to capture the energy from those moving cars, and convert and store it as electricity?
  - But how?



PIEZOELECTRIC MATERIALS



- The word "piezoelectricity" means electricity generated from pressure.
- Piezoelectric materials generate internal electrical charge under the influence of mechanical force.
- Direct Piezoelectric Effect:
  - The application of compression or tension stress on a piezoelectric material creates a voltage.

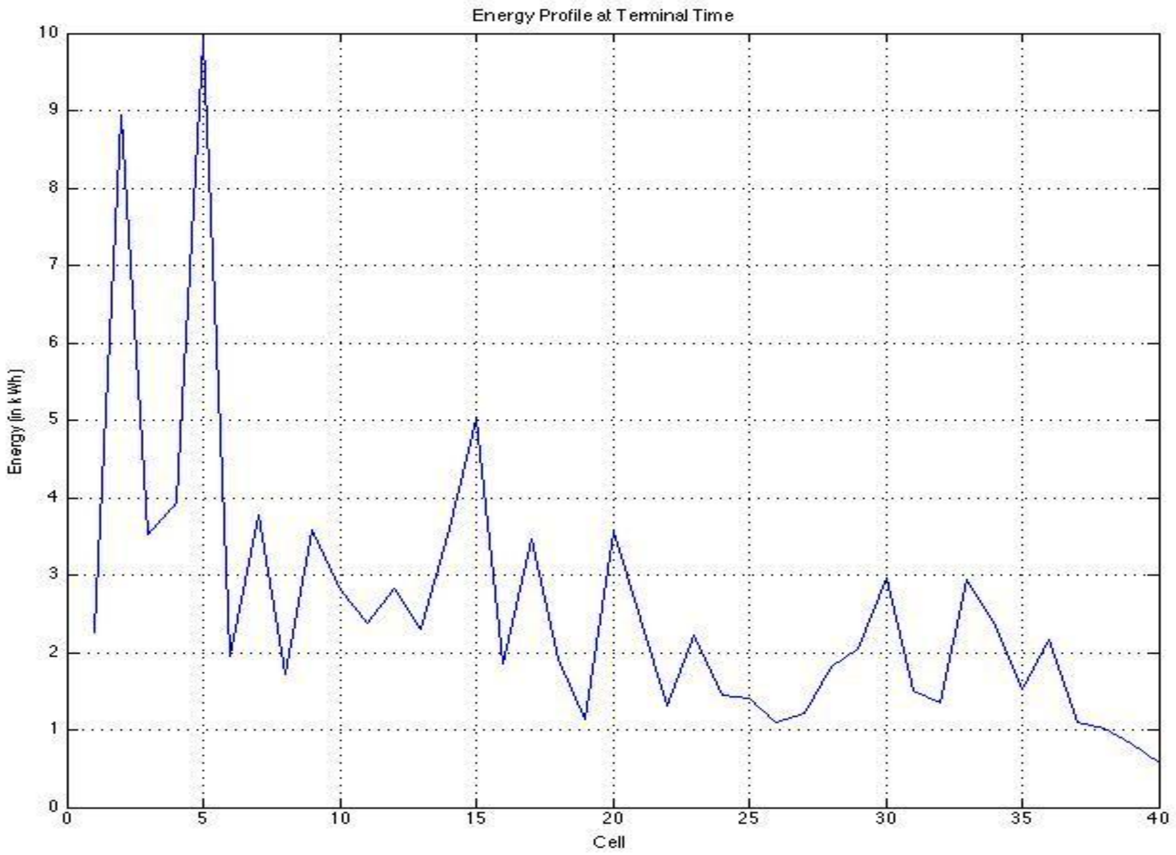


How we use LWR and TOPL to run our stimulation

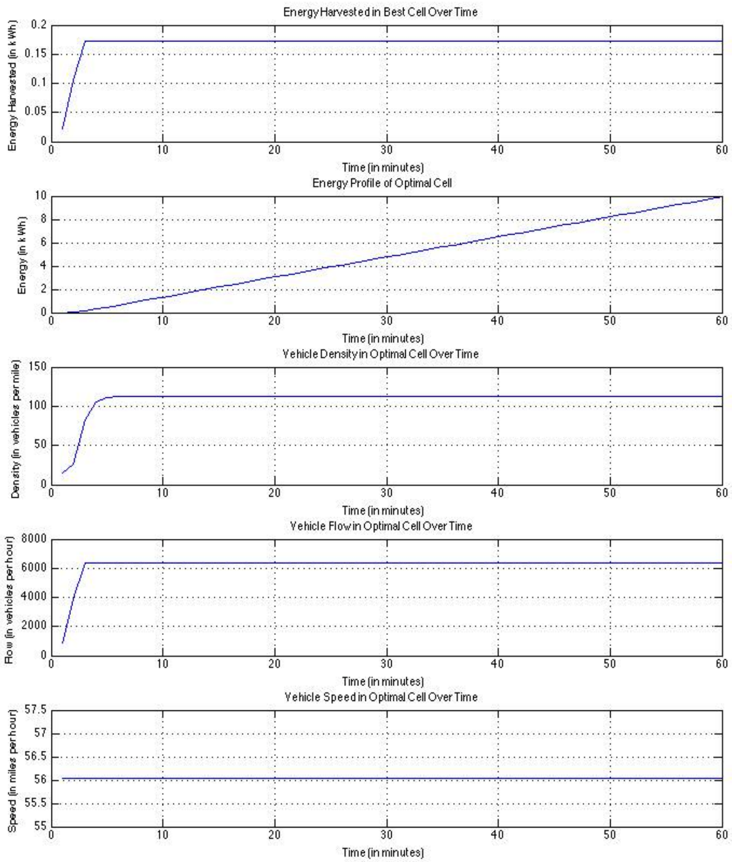
- TOPL uses LWR to define data such as critical density.
- By using numerical data provided by TOPL, we are able to run a stimulation and figure out the optimized cell (which generates the most energy)

Numerical Results

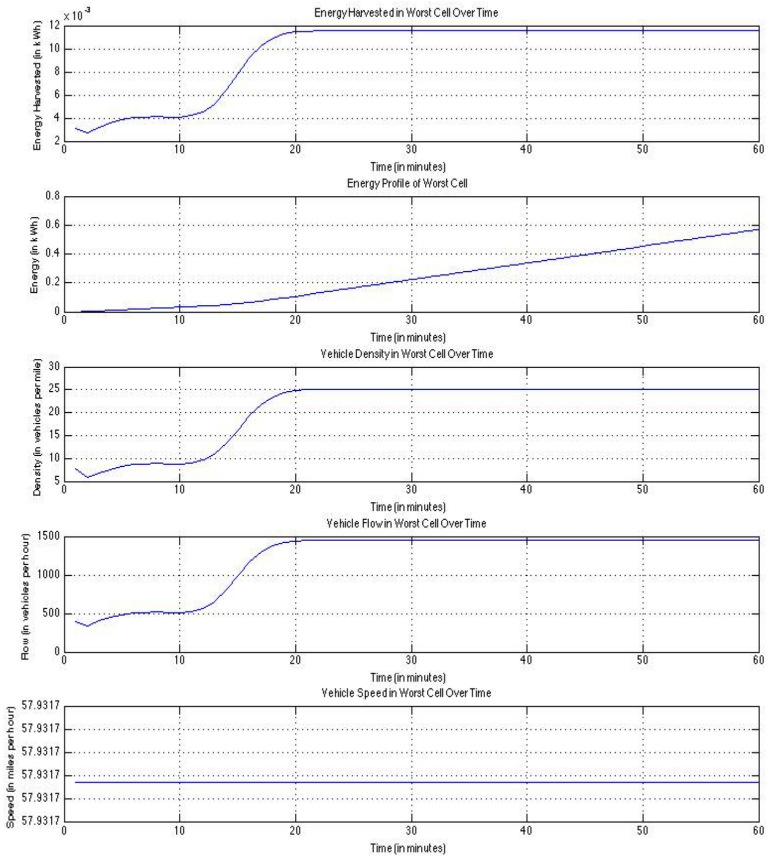
- Tile length: 20 meters ($665/meter)
- Flow to energy mapping:
  - For 1 km tile, 1 veh/hr → 2/5 kWh/hr
- As of Dec 2011, cost of electricity = $0.12/kWh



NUMERICAL RESULTS



- Best Cell: 5
  - Capacity = 6807 vph
  - Critical density = 121.5 vpm
  - On-ramp flow = 12 vph



- Worst Cell: 40
  - Capacity = 9327 vph
  - Critical density = 161 vpm
  - On-ramp flow = N/A

Comparison

- Solar energy at peak sun
  - 0.17 kWh/ (hr*m^2)
- Piezoelectric Tiles
  - 0.5 kWh/ (hr*m^2)

--End--

Thank You!