Reachability/Unsafe Region Analysis for Drifters

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Presentation Outline

• Introduction
• Motivation and Problem Statement
• Mathematical Tools
• Implementation
• Results
Introduction

http://float.berkeley.edu/drifters
Motivation

• A network of drifters in a river can provide real-time updates on river conditions.
• Long term operation requires avoiding obstacles and collision with the shore.
Problem Statement

• Regions of unsafe operation must be determined in order to prevent collisions between the drifter and the shore.
Mathematical Tools

• Consider a simple dynamical system with bounded input

\[
\begin{align*}
\dot{x} &= a_x(t) \\
\dot{y} &= a_y(t) \\
\|a\|_2 &\leq \bar{a}
\end{align*}
\]

• How do we define reachability?
  • Assume \(x_0 = (0,0)\), then reachable set is the set of states such that there exists a sequence of actions \(a(t)\) to maneuver the system to that set
  • For this system, \(R^2\) is reachable
Mathematical Tools

- Minimum-Time-to-Reach is an extension upon this idea
- By the time $t = \tau$, what states are reachable?
Mathematical Tools

- Backward Reachability – Modify model to include constant disturbance

\[
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} = \begin{bmatrix} a_x(t) \\
a_y(t) \end{bmatrix} + \begin{bmatrix} 0.75 \\
0.5 \end{bmatrix}
\]

- Define a target set, \( T \)

\[
T = \{(x, y) | x^2 + y^2 \leq 1\}
\]

- From what states can we guarantee controllability into \( T \)?
Mathematical Tools

- From what states can we guarantee controllability into $T$?
Implementation

• We start with a dynamics model

\[ \dot{x} = f(x, a, b) \]

• \( x \) is the state

• \( a \) defines drifter inputs

• \( b \) defines disturbance inputs

\[
\begin{align*}
\dot{x} &= w_x(x, y) + a_x(t) + b_x(t) \\
\dot{y} &= w_y(x, y) + a_y(t) + b_y(t) \\
\|a\|_2 &\leq \bar{a}, \quad \|b\|_2 \leq \bar{b}
\end{align*}
\]
Hamilton-Jacobi-Isaacs PDE

- Use numerical solver (Level Set Toolbox) to solve HJI PDE for $v(x, t)$:

$$v_t(x, t) + \min[0, H(x, v_x(x, t))] = 0$$

- Need to define a target set, $T$, and Hamiltonian

  - $v(x, 0) = v_0(x) = \begin{cases} -1, & \text{for } x \in T \\ +1, & \text{otherwise} \end{cases}$

  - $H(x, p) = \max_{\|a\|_2 \leq \bar{a}} \min_{\|b\|_2 \leq b} p^T f(x, a, b)$
Target Set Definition
Hamiltonian

\[ H(x, p) = \max_{\|a\|_2 \leq \bar{a}} \min_{\|b\|_2 \leq \bar{b}} \ p_1 [w_x + a_x + b_x] + \]

\[ p_2 [w_y + a_y + b_y] \]

\[ = p_1 w_x + p_2 w_y + [\bar{a} - \bar{b}] \sqrt{p_1^2 + p_2^2} \]
Flowfield Data
Visualization of $\nu(x, t)$
Time-varying flowfield
No Inputs – Free Drift
Comparisons
Simulated Trajectories
Conclusion

• This method has seen actual use in determining safe/unsafe regions for determining control strategies

• Modeling Assumptions
  • Unrealistic model

• Visualize safe and unsafe trajectories
References

• Mitchell, I. A Toolbox of Level Set Methods. 1 June 2007. UBC.
• Bayen, A. ME 236/EE 291 Course Notes.
• REALM, River Estuary and Land Model.
Thanks!