Highway Traffic Density Estimations Informed by Velocity Measurements

Matt Wright and David Shulman
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Outline

- Motivation
  - Methods of flow measurement
- Computational Model Theory
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- Algorithm Test Case
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  - Site Description
  - Test Cases

Motivation

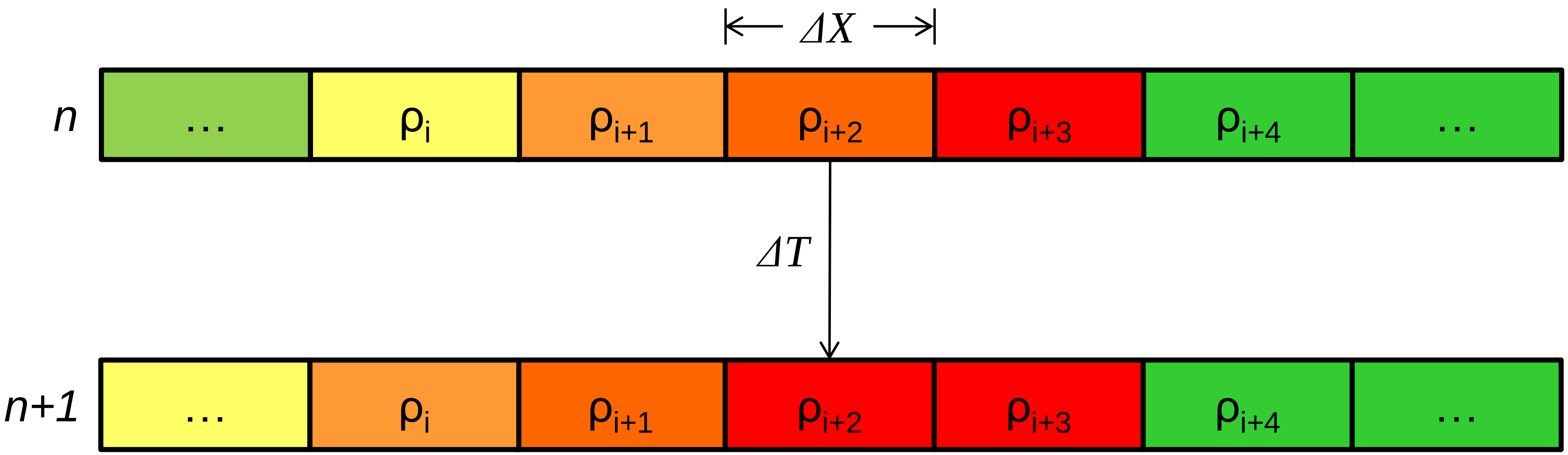
- Current method of flow measurement
  - Inductive loops
  - Limitations

- Additional Information
  - GPS position data
  - Calculated velocities

Computational Model Theory

Cell Transmission Model (CTM)

- Discrete representation of linear traffic flow in space and time
- Longitudinally adjacent cells with mean densities



$$\rho_i^{n+1} = \rho_i^n - \frac{\Delta T}{\Delta X}\left(G(\rho_i^n, \rho_{i+1}^n) - G(\rho_{i-1}^n, \rho_i^n)\right)$$

Computational Model Theory

Godunov Discretization

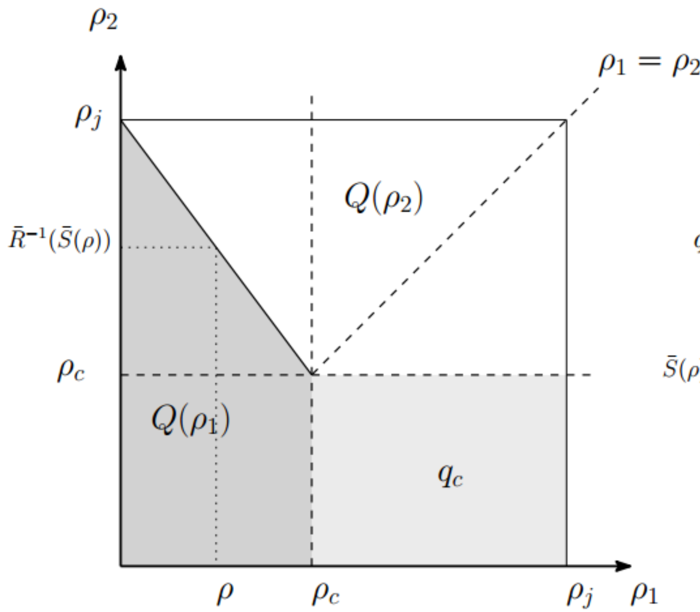
- Determines flux between cells
- Evaluates minimum of upstream sending flow and downstream receiving flow

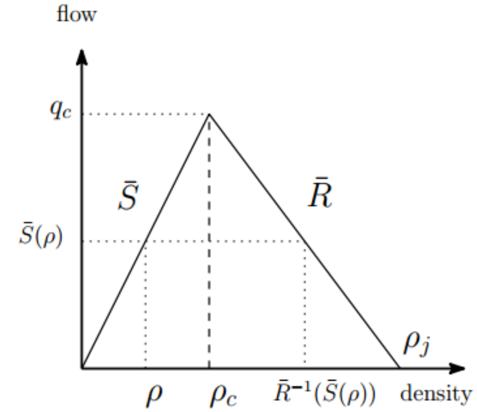
General Form

$$G(\rho_1, \rho_2) = \begin{cases} min_{\rho \in [\rho_1, \rho_2]} Q(\rho) & if \quad \rho_1 \leq \rho_2 \\ max_{\rho \in [\rho_2, \rho_1]} Q(\rho) & if \quad \rho_2 \leq \rho_1 \end{cases}$$

$$G(\rho_1, \rho_2) = \begin{cases} Q(\rho_2) & if \quad \rho_c \leq \rho_2 \leq \rho_1 \\ q_c & if \quad \rho_2 \leq \rho_c \leq \rho_1 \\ Q(\rho_1) & if \quad \rho_2 \leq \rho_1 \leq \rho_c \\ \min(Q(\rho_1), Q(\rho_2)) & if \quad \rho_1 \leq \rho_2 \end{cases}$$

Computational Model Theory

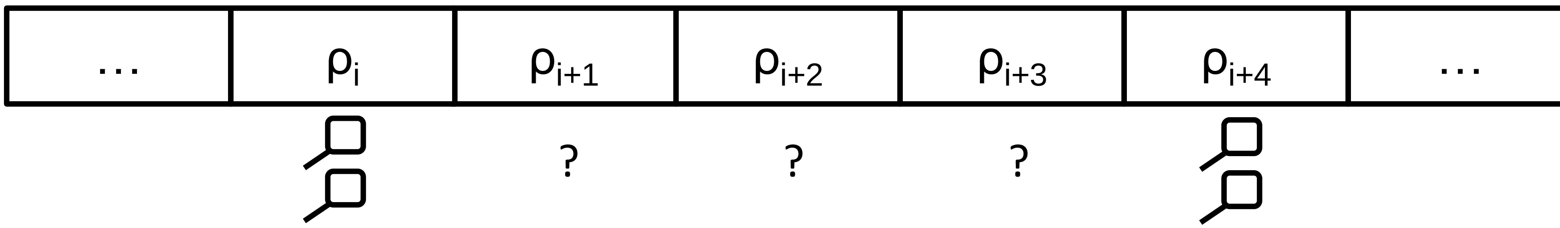




Flux =	Greenshields	Daganzo-Newell	Condition
$G(\rho_1, \rho_2) =$	$\begin{cases} v_{max}\left(\rho_2 - \frac{\rho_2^2}{\rho_j}\right) \\ q_c \\ v_{max}\left(\rho_1 - \frac{\rho_1^2}{\rho_j}\right) \end{cases}$	$\begin{cases} w_f(\rho_2 - \rho_j) \\ q_c \\ v_{max}\rho_1 \end{cases}$	$(\rho_1, \rho_2) \in$ *white region* $(\rho_1, \rho_2) \in$ *light grey region* $(\rho_1, \rho_2) \in$ *grey region*

Computational Model Theory

Incorporating Real-World Data



Greenshields

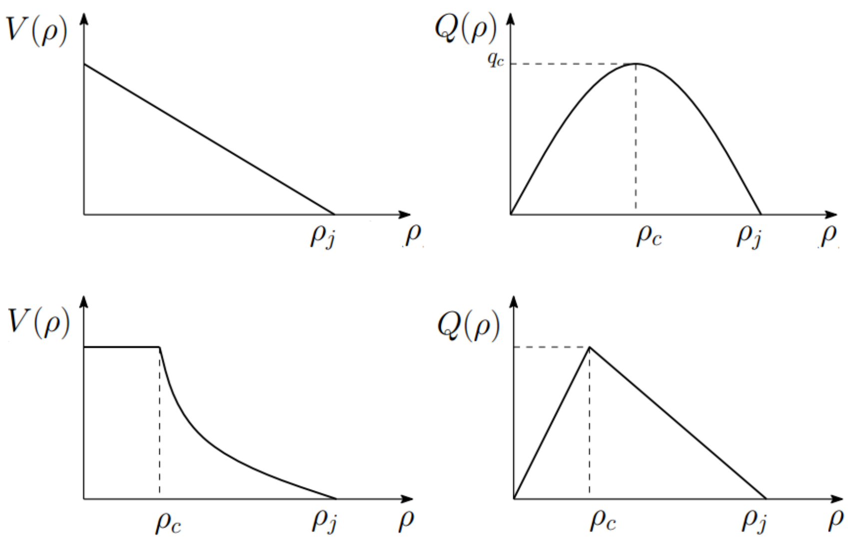
$$v = v_{max}\left(1 - \frac{\rho}{\rho_j}\right)$$

Daganzo-Newell

$$v = \begin{cases} v_{max} \\ w_f\left(1 - \frac{\rho_j}{\rho}\right) \end{cases}$$

Velocity

Flux



Computational Model Theory

Ensemble Kalman Filtering (EnKF)

- Provides feedback to adjust model
- Estimates states in regions that have lack data

$$\begin{bmatrix} \rho_f^n(k) \\ v_f^n(k) \end{bmatrix} = M \begin{bmatrix} \rho_a^{n-1}(k) \\ v_a^{n-1}(k) \end{bmatrix} + \eta^n(k)$$

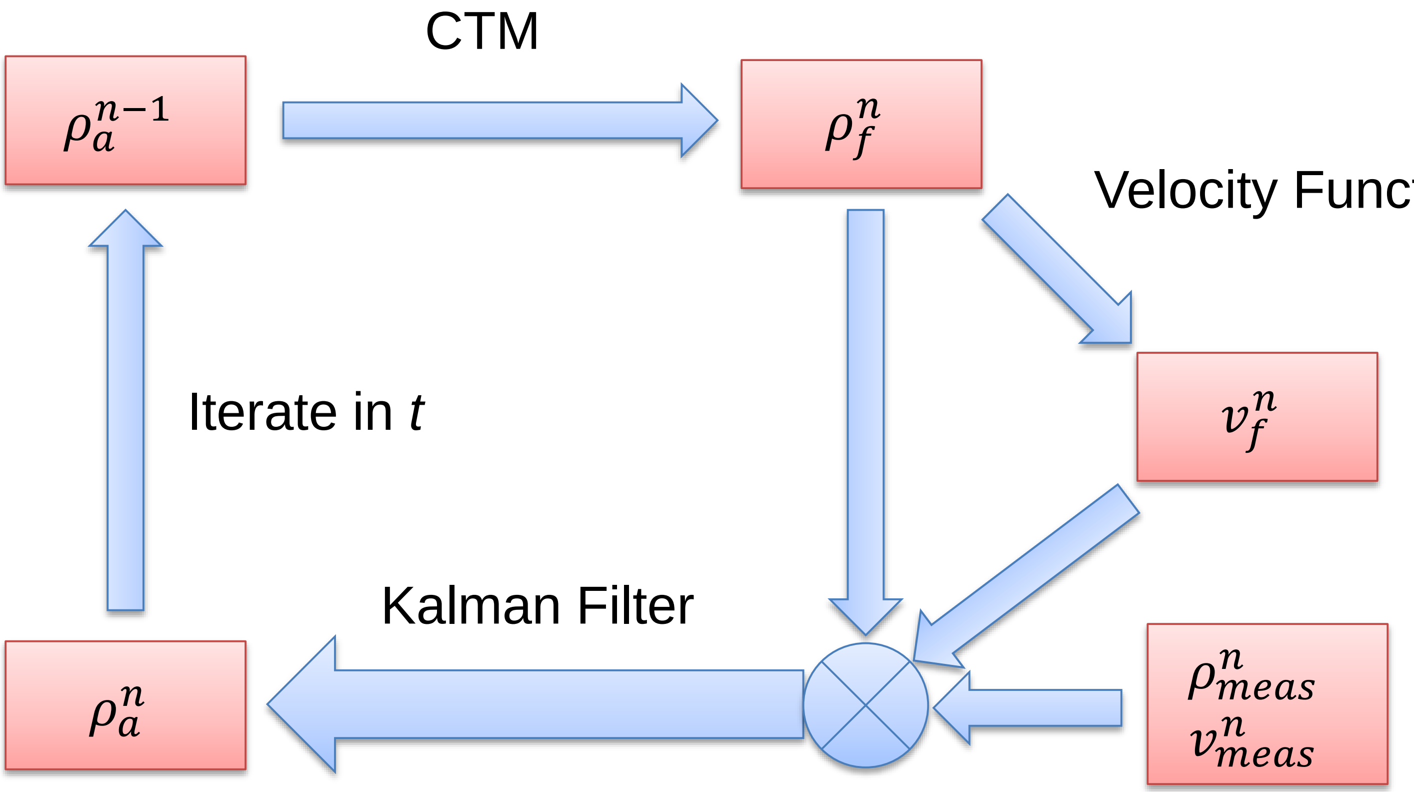
$$\begin{bmatrix} \bar{\rho}_f^n \\ \bar{v}_f^n \end{bmatrix} = \frac{1}{K} \sum_{k=1}^{K} \begin{bmatrix} \rho_f^n(k) \\ v_f^n(k) \end{bmatrix}$$

$$P_{ens,f}^n = \frac{1}{K-1} \sum_{k=1}^{K} \begin{bmatrix} \rho_f^n(k) - \bar{\rho}_f^n \\ v_f^n(k) - \bar{v}_f^n \end{bmatrix} \begin{bmatrix} \rho_f^n(k) - \bar{\rho}_f^n \\ v_f^n(k) - \bar{v}_f^n \end{bmatrix}^T$$

$$G_{ens}^n = P_{ens,f}^n (H^n)^T (H^n P_{ens,f}^n (H^n)^T + R^n)^{-1}$$

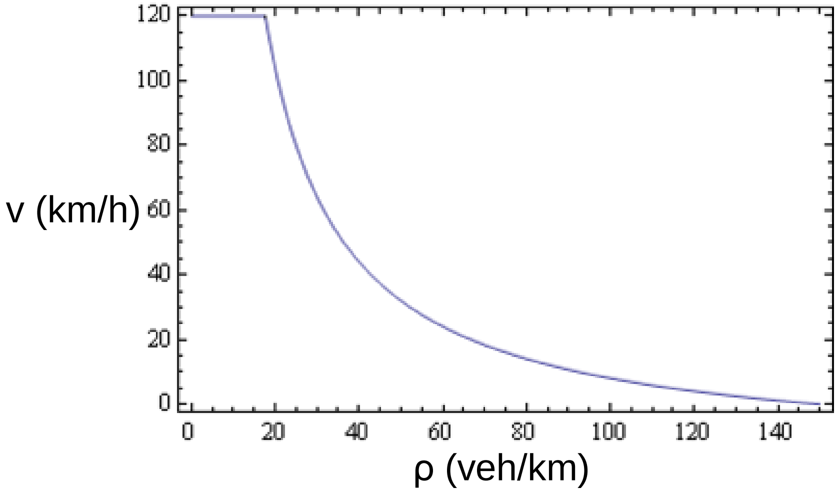
$$\begin{bmatrix} \rho_a^n(k) \\ v_a^n(k) \end{bmatrix} = \begin{bmatrix} \rho_f^n(k) \\ v_f^n(k) \end{bmatrix} + G_{ens}^n \left(\begin{bmatrix} \rho_{meas}^n \\ v_{meas}^n \end{bmatrix} - H^n \begin{bmatrix} \rho_f^n(k) \\ v_f^n(k) \end{bmatrix} + X^n(k) \right)$$

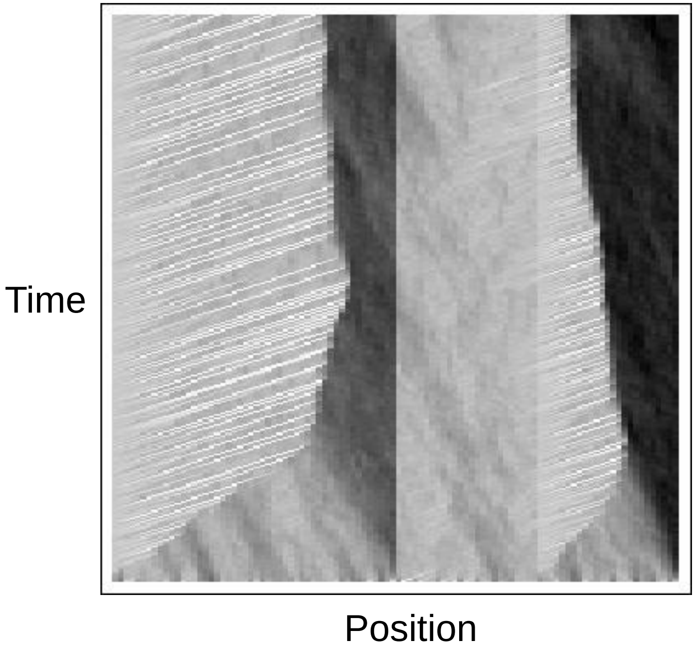
Model Summary

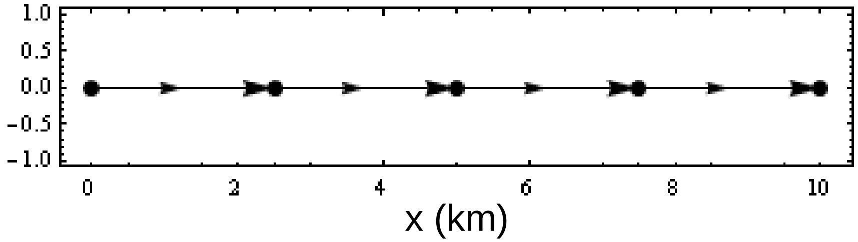


Algorithm Test Case

- Mathematica Reference Implementation
- Algorithmically identical to actual system for live data
- Made for debugging purposes







Algorithm Test Case

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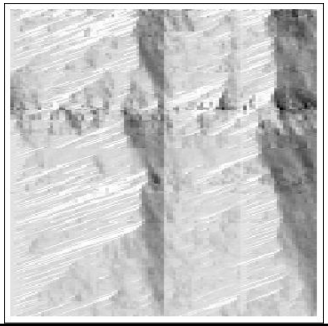
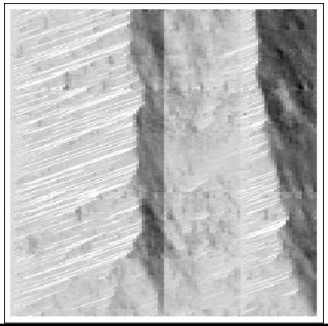
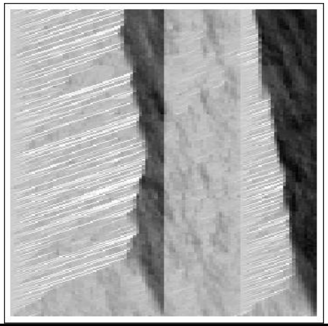
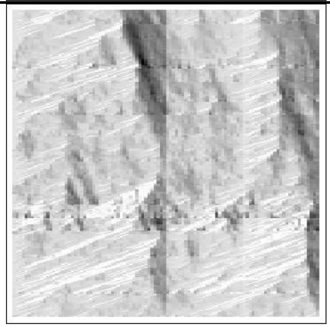
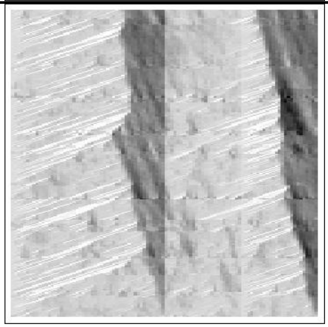
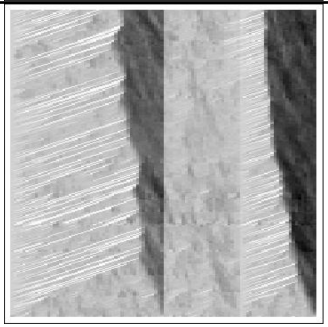
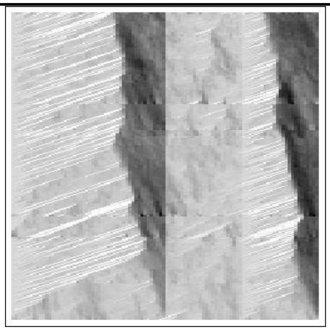
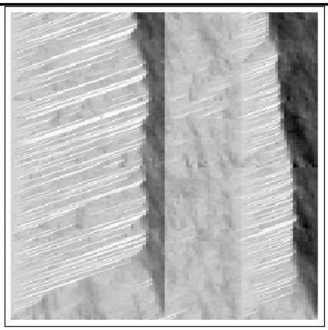
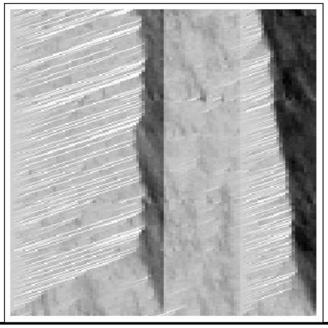
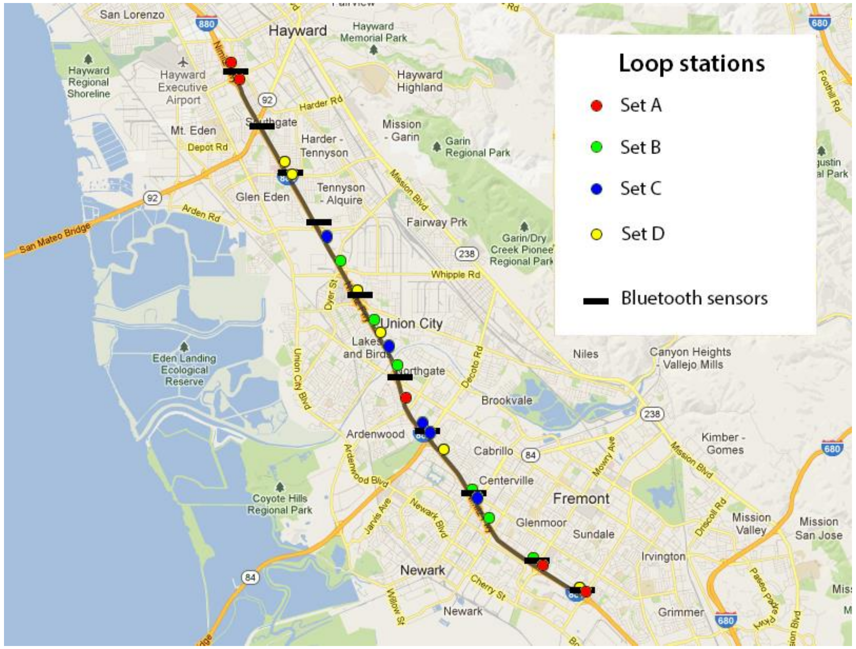
0%			
5%			
15%			

Future Work: Real Data Problem

- Site
  - 11-mile stretch of I-880
  - NB Lane only
- Data availability
  - 10 stationary loop detectors
    - For comparison against estimated density
  - ~10000 probe points



Future Work: Real Data Test Cases

Test	Purpose
Sequentially remove loops, compare estimate to truth	Test probe data at filling in single-loop gaps
Stochastically remove several loops	Test error variance for particular loop removal
Include only loops upstream of congestion events	Test probe data's usefulness at finding end of congestion events

- Several test cases proposed/underway
- Repeated for varying amounts of sampled probe points
- Estimation error can be evaluated visually or numerically

Thank You

Questions?