# CE 291F, ME 236, EE 291
# Control and optimization of distributed parameters systems

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| **Course No.:** | CE 291F, ME 236, EE 291 |
| **Course name:** | Control and optimization of distributed parameters systems |
| **Transcript Title:** | Opt control of PDEs |
| **Course Format:** | Two 1½ hour lectures per week. |
| **Prerequisite:** | E77, MA 54 (or equivalent), or instructor consent. |
| **Lectures:** | TuTh: 2:00-3:30, 406 Davis Hall |
| **Office Hours:** | 711 Davis Hall, TBD |
| **Grading:** | Letter |

**Remarks:** Students will learn the basic concepts and methods for solving simple partial differential equations analytically and numerically. They will also learn fundamental control techniques that can be applied to these PDEs. Applications to various CEE problems and real data will be developed by the students through class project that will start in the middle of the semester.

**Catalog description:** Distributed systems and PDE models of physical phenomena (propagation of waves, network traffic, water distribution, fluid mechanics, electromagnetism, blood vessels, beams, road pavement, structures, etc.). Fundamental solution methods for PDEs: separation of variables, self similar solutions, characteristics, numerical methods, spectral methods. Stability analysis. Adjoint-based optimization. Lyapunov stabilization. Differential flatness. Viability control. Hamilton-Jacobi based control.

**Expanded description:** Attached

**Required Textbook:** A reader will be prepared. There does not exist a textbook covering all the topics above, that would be accessible to students.

**Recommended Textbook:** None.

**Course objectives:** Introduce the students to the concepts and methods of modeling, control and optimization of systems driven by partial differential equations (i.e. distributed parameters systems).

**Instructor in charge:** Alexandre Bayen

## EXPANDED DESCRIPTION
## Control and optimization of distributed parameters systems

**Basic Philosophy:** Because of their complexity, some engineering phenomena or systems are best described by partial differential equations. The control of these phenomena thus cannot rely on traditional control and optimization methods. This class is an introduction to the fundamental techniques used to model, control and optimize such systems.

**Remark:** This class targets two categories of engineering students. The first category is students with a partial differential equations background. Usually, these students do not have prior exposure to control. The second category consists of students with a good background in control, but who usually do not know partial differential equations. This class will be designed to be accessible to both communities.

**Objectives:**

- Learn fundamental modeling techniques to describe distributed parameter systems with partial differential equations. This part of the class will be novel for students without exposure to partial differential equations. Examples will be used from several areas of CEE, ME and EECS, to illustrate the usefulness of PDEs for describing complex phenomena such as heat transfer, contaminant propagation in water, vibrations of beams and membranes, propagation of signals in networks, highway transportation problems.
- Learn fundamental techniques that can be used to control systems driven by partial differential equations. The methods will be devised in the specific context of the applications, so that the students do not have to learn the full theory, but only the important features for their applications.
- Learn the different computational techniques that can be applied to solve these problems numerically. Have an idea of the complexity of the solutions methods required by engineering problems arising in this field.
- Do a small literature search on a topic chosen by the student in a list proposed by the instructor. This part can either be viewed as a first research experience for a student interested to do a PhD in this field, or as a review that an engineer would have to do to figure out the existing results that he/she could apply to solve a given problem.

**Summary of topical outline:**

- Introduction to partial differential equations (2 weeks)
- Self-similar solutions of PDEs (1 week)
- Separation of variables, eigenvalue decomposition (2 weeks)
- Characteristics (1 week)
- Conservation laws (1 week)
- Periodic solutions, stability (1 week)
- PDE motion planning using differential flatness (1 week)

- Adjoint-based methods (2 weeks)
- Lyapunov stability, energy methods (1 week).
- Viability techniques, Hamilton-Jacobi techniques (1 week)
- Research projects, student presentations, lab (2 weeks)

**Expanded topical outline:**

- Introduction to partial differential equations (2 weeks)
  - Partial derivatives
  - Differential operators
  - Variable changes
  - Derivation of PDEs from fundamental principles
  - The LWR equation for highway traffic, the heat equation
  - Solutions of PDEs, initial conditions, boundary conditions, existence, uniqueness
- Self-similar solutions of PDEs (1 week)
  - The Vashy-Buckingham Pi theorem
  - Application: atomic explosions
  - Transforming PDEs into ODEs via self-similar variables
- Separation of variables, eigenvalue decomposition (2 weeks)
  - General methodology, series decomposition
  - Application: vibration of a string
  - Cylindrical membranes
  - Particular solutions
  - Splitting
- Characteristics (1 week)
  - Characteristics for first and second order systems
  - Illustration for highway traffic
  - Illustration for channel flow: characteristics of the Saint-Venant equations. Subcritical/supercritical flows.
- Conservation laws (1 week)
  - Rankine Hugoniot jump conditions
  - Application for highway traffic: shockwaves on the highway
  - Other applications: supersonic flows, waves in water channels
- Periodic solutions, stability analysis (1 week)
  - Periodic solutions, complex embedding
  - Dispersion relations for nonlinear PDEs
  - Stability analysis
- PDE motion planning using differential flatness (1 week)
  - Motion planning: explicit solutions
  - Input/output control
  - Illustration: thermal control of a beam, control of highway congestion, shockfree control of PDEs
- Adjoint-based methods (2 weeks)
  - Definition of adjoint-based optimization
  - Computation of the gradient

    - An extended example: network of PDEs applied to highway control
- Lyapunov stability, energy methods (1 week).
    - Energy methods: definition of Lyapunov stability in the context of PDEs.
    - Stabilization control.
    - Application: stabilization of beams and structures.
- Viability techniques, Hamilton-Jacobi techniques (1 week)
    - Implicit surfaces, level sets
    - Control of safety sets with Hamilton-Jacobi equations
- Research projects, student presentations, lab (2 weeks)
    - Presentation of students: literature reviews, projects.

**General information**

- There will be 4 homeworks. The homework will span the following four topics: self-similar solutions, separation of variables, characteristics, conservation laws. This will ensure that the students have the required material for being able to devise appropriate control strategies for the systems of interest.
- There will be a midterm, open-book, and open notes.
- There will be one class project. Students are encouraged to bring their own research-related projects. Projects will be suggested to students, if they need. Students are allowed to team up for projects, if the scope of the project is large.

**Grading**

The final grade will be determined based on the performance on homework assignments, the midterm, laboratory results and final report. Homeworks will include the possibility to extend the results derived in class to gain additional credit. Homework, midterm and lab will be weighted as follows:

Homework: 40% (4 at 10%)
Midterm: 20%
Lab: 40%

**Laboratory**

For the project, the students will be expected to conduct significant work on a topic of their choice. Projects will be suggested in the following topics:

- Modeling systems with PDEs for control purposes
- Algorithm design for control and/or optimization of PDE driven systems
- Simulation tools for control of PDE driven systems
- Hardware implementation of control and/or optimization algorithms on a PDE driven system

Each project will start with a review of the literature on the subject. If the students choose their own research topics, they will be responsible for finding the proper set of articles relevant for the problem considered. If they choose one suggested project, some references will be provided as a basis for further reading. Depending on the topic, they will balance their time between algorithm design, simulation, and/or hardware implementation. In the first weeks of the project, they will be expected to set up clear goals with the instructor, and a plan to achieve these goals. They will meet with the instructor several times to assess the progress made on the project. They will give a short presentation of your project to the class at the end of the semester.

**Reporting**

Students will be expected to write a report, to summarize their work. The report should be written in a way which is understandable for someone who does not have exposure to the field. Here is a rough outline of what would be expected from a student:

1. Introduction: objectives of the project, background and motivation.
2. Literature review: describe the state of the art in the field; include all proper references, explain where the project fits.
3. Problem investigated. Describe the physical system modeled, eventually describe the derivation of the model. Pose the problem of controlling the system.
4. Control. If the student is deriving his/her own control or optimization algorithm, include all derivations. Summarize theoretical contributions.
5. Simulation. If the student is designing a simulation tool for control or optimization purposes, describe which algorithms are used, and how they address the problem. Describe the software implementation, and the validation of the software (for example on model problems).
6. Hardware implementation. If the student is implementing control algorithms on an experimental testbed, describe which algorithms he/she have used, and how they address the problem. Describe the hardware implementation.
7. Results: present the results of experiments in tabular and/or graphical form, but include text that organizes and describes the results to guide the reader through them.
8. Discussion: discuss the results, compare with theory, comment on the significance of the results, discuss reasons for disagreement, and suggest how the measurements and the experiment could be improved.
9. Summarize the main results and findings of the experiment
10. Bibliography: list all references used in the text.
11. Appendix: include derivations, raw data, calculations, and spreadsheets if appropriate.