The Evolution of
2D-Implicit Surfaces
Using Level Set Methods

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Presentation Outline

• Motivation:
  – waste package in convective body of water

• Implicit Surfaces & Level Set Methods
  – Definitions, Introduction

• Numerical Solution to Hamilton-Jacobi (HJE) PDE
  – Assumptions, Boundary Conditions

• Included Types of Motion:
  – Convection, Rotational, Normal, Mean Curvature

• Preliminary Results
• Future Work
• Acknowledgements / References
Level Sets & Implicit Surfaces

- Implicit, rather than Explicit Surface:
  \[ \psi : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R} \]

- Level Set Function -> Isosurfaces
  \[ \phi(x,t) : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R} \]

- Allows for surface separation:

- Intuitive, but high computation cost

- Numerically computed for this project

HJE with 4 terms

Hamilton Jacobi PDE:

for \( x \in \mathbb{R}^n, t \geq 0 \)

\[
D_t \phi(x,t) + v(x,t) \cdot \nabla \phi(x,t) + a(x,t) \| \nabla \phi(x,t) \| - b(x,t) \kappa(x,t) \| \nabla \phi(x,t) \| = 0
\]

Partial Time Derivative | Convection Term (Velocity Field) | Normal Direction Motion | Mean Curvature Motion

Subject to initial conditions: (zero level set isosurface)

\[ \phi(x,t) = \phi_0(x) \quad \text{for} \quad x \in \mathbb{R}^n \]
**Convection & Rotation**

\[ \nu(x) \cdot \nabla \phi(x,t) \]

- Velocity Field (vector)
- Gradient of Level Set

Transformation Matrix:
- Linear Motion
  \[
  \begin{bmatrix}
  1 & 0 \\
  0 & 0 \\
  \end{bmatrix}
  \]
- Rotation
  \[
  \begin{bmatrix}
  0 & 1 \\
  -1 & 0 \\
  \end{bmatrix}
  \]

**Normal Motion Tern**

\[ a(x) \| \nabla \phi(x,t) \| \]

- Normal Speed
- Gradient of Level Set

(motion out of the page)
**Mean Curvature Term**

\[ b(x) \kappa(x,t) \| \nabla \phi(x,t) \| \]

Surface Speed \quad Mean Curvature \quad Gradient of Level Set

**Modeling Assumptions**

- Implicit Surface is continuous: \((\text{zero level set})\)
  \[ \psi(x,0) = \psi(x) \rightarrow \phi_0 \]

- Diffusion Effects \ll Convection Effects
- Non-Rigid Body (can deform)
- Mixing effects w/ medium are negligible
- Time independent velocity field:
  \[ v(x,t) \rightarrow v(x) \]
Convection Results

Rotation Results

\[ \text{tmax} = 1.2 \]
\[ \text{tstep} = 0.2 \]
\[ \text{t0} = 0 \]
\[ V = 1 \]

\[ \text{rotation} = -0.075\pi \]
Normal Motion Results

\[ \begin{align*}
\text{tmax} &= 1.2 \\
\text{tstep} &= 0.2 \\
\text{t0} &= 0 \\
\text{aValue} &= 0.20 \\
\text{useTimeDependent} &= 0
\end{align*} \]

Mean Curvature Results

\[ \begin{align*}
\text{tmax} &= 1.2 \\
\text{tstep} &= 0.2 \\
\text{t0} &= 0 \\
\text{bValue} &= 0.02 \\
\text{useTimeDependent} &= 0
\end{align*} \]
Combined Results...

How about real data?

Velocity Field, Sacramento Georgiana Slew (courtesy Jean Severin)
2D Velocity Field

Mesh of Velocity Field
3D Interpolated View

Velocity Field, Sacramento Georgiana Slew (courtesy Jean Severin)

Limitations / Future Work

- Implementation of Velocity Field
  - Interpolation of all points finished
  - Implement in way that Toolbox can read
  - Reduce Size of Grid (i.e. only part of slew)

- Define more realistic implicit surface
  - i.e. amorphous plumes instead of circles and stars

- Implement real constants / slew parameters
  - Mean curvature terms, rotational speed, etc.

- Distribute computation (parallel computing)?
  - Allows for higher resolution, more complex environments

- Possibly add Separation term to HJE?
  - Contaminants can separate: viscosity and surface tension
References

A. Bayen, Personal Communication


J. Severin, Personal Communication (Georgiana Slew data)

*Mean Curvature Motion*. http://iria.pku.edu/~jiangn/courses/IRIA/node172.html.