“The Generalization and Application of Particular Solutions to Lamb’s Problem using Fourier Analysis and System Identification Techniques”

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Outline

- Lamb’s Problem:
- Background:
  Wave Propagation in Elastic Solids
- Solution Strategies:
  Analytical vs Numerical
- Formulation of the Problem
- Preliminary Results
Lamb’s Problem

“Tremors Over the Surface of an Elastic Solid” (1904)

Why is Lamb’s Problem Important?

Wave Propagation in Elastic Solids

Waves in an infinite solid

\[(\lambda + \mu)u_{,ij,ii} + \mu u_{,ii} + \rho f_i = \rho u_i\]

\[(\lambda + \mu)\nabla \nabla \cdot \mathbf{u} + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f} = \rho \ddot{\mathbf{u}}\]

indicial notation

vector notation
Wave Propagation in Elastic Solids

Types of Waves in Solids

• Pressure Waves
  – Dialational
  – Only volume change

• Shear Waves
  – Distortional
  – No Volume Change

• Surface Waves
  (Rayleigh Waves)

Source:
“Acoustics Animations”
Dr. Dan Russell
Kettering University Applied Physics

State-of-the-art solution strategies

• Analytical
  – Special cases only
  – Integrals!!!
    • Laplace, Fourier methods
    • Complex contours of integration subdivided around branch points and poles

• Numerical –
  – VERY computationally intensive!
    • Start with the underlying PDE and go with it…
    • Every point must be solved for (hard to simplify the problem)

  – Finite difference
  – Finite element
    • Explicit formulation
    • Instabilities
    • Element Instabilities
State-of-the-art solution strategies

**Analytical Realm**

- PDE
  - Simplified PDE for special case physics
  - Analytical Solutions for Special Locations and specific Material Properties
  - Specific Analytical Solution (for toy problems)

**Numerical Realm**

- General Problems
  - Computational Errors
    - High Computation Costs (problems must be small in size)
  - General Solution to a small problem (from a black box)

Use Superposition and Transform Methods

General Solution with fewer errors and no black box
Formulation of the Problem

Differential Flatness: (output as a function of the input)

- **Classical Problem:** (construct an input tailored to produce the required output)

  ![Diagram](Diagram.png)

- **Inverse Modeling:** (predict the input which was required to produce the observed output)
  (Estimation)

  ![Diagram](Diagram.png)

Lamb’s Problem

“Tremors Over the Surface of an Elastic Solid” (1904)

*Horace Lamb*

http://www.history.nesc.ac.uk/~hdbisplay/Lamb.html

Why is Lamb’s Problem Important?
Fourier’s Theory: Superposition of Sinusoids

Source: “Acoustics Animations”
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Superposition of Solutions to Sinusoidal Loading

800 terms
1600 terms
4000 terms
Problem! Gibbs phenomina

- Truncation of an infinite series of sinusiods crease ripples.

Due to Gibbs Phenomenon, high frequency ripples in the forcing function inadvertently create transient waves which are purely an artifact of the superposition of discrete signals and “swamp out” the transient wave of interest.

Pekeris “The Seismic Surface Pulse” 1959

- Analytical solution for the surface of a halfspace with a step function transverse surface loading

Solution with respect to tau (a self similar variable) -- also characteristics.
Next Idea

- I would like to obtain the “transfer function” for the material response. (Impulse response function)
- The derivative of a step is an impulse
- Use Transform Theorems to numerically compute the non-dimensional impulse response function for the transverse surface displacements.

Once the impulse response function is obtained, standard differential flatness-based steering and control (prediction) can be implemented.
Questions?