Using wave propagation properties to identify soil parameters

Adjoint based method and optimization algorithms

1-D Equation of Propagation of a shear wave in soil

\[
\rho(x) \frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial}{\partial x} \left( g(x) \frac{\partial u(x,t)}{\partial x} \right) + \frac{\partial}{\partial x} \left( \eta(x) \frac{\partial^2 u(x,t)}{\partial x \partial t} \right)
\]

Distributed Mass model

- Boundary conditions:
  \( u(0,t) = \) input signal
  \( u(L,t) = \) output signal

- Initial conditions:
  \( u(x,0) = 0 \)
  \( \frac{\partial}{\partial x} u(x,0) = 0 \)
Cost function - Adjoint based method

\[ J = \frac{1}{2} \int_{x,T} \left[ u^{obs}(x,t) - u(x,t) \right] dt dx \]

\[ \rho(x) \frac{\partial^2 q(x,t)}{\partial t^2} = \frac{\partial}{\partial x} \left[ g(x) \frac{\partial q(x,t)}{\partial x} \right] - \frac{\partial}{\partial x} \left[ \eta(x) \frac{\partial^2 q(x,t)}{\partial x \partial t} \right] + \sum \left[ u^{obs}(x,t) - u(x,t) \right] x = x_f \]

Boundary conditions:
\[ q(0,t) = 0 \]
\[ q(L,0) = 0 \]
\[ \frac{\partial}{\partial x} q(L,0) = 0 \]

Initial conditions:
\[ q(x,T) = 0 \]
\[ \frac{\partial}{\partial t} q(x,T) = 0 \]
\[ \frac{\partial}{\partial x} q(x,T) = 0 \]

The Gradients

\[ \frac{\partial J}{\partial g} = \int_T \frac{\partial q}{\partial x} \frac{\partial u}{\partial x} dt \]

\[ \frac{\partial J}{\partial \eta} = \int_T \frac{\partial q}{\partial t} \frac{\partial^2 u}{\partial x \partial t} dt \]
Methodology

1. Start with initial guess $g_0, n_0$
2. Solve the PDE (2) of $u$ with $g_0, n_0$
3. Solve the Adjoint PDE (2) for $\phi$
4. Calculate the gradient $\nabla J$
5. Update $g_{n+1}, n_{n+1}$
6. Evaluate $J$
7. Iterate

- If $J < J_0$ then $\text{YES, STOP}$
- Else $\text{NO, Iterate}$

First Results

Input Signal

Output Signal
Multi-layer problem

\[
\begin{align*}
\rho_1(x) \frac{\partial^2 u_1(x,t)}{\partial t^2} &= \frac{\partial}{\partial x} \left( g_1(x) \frac{\partial u_1(x,t)}{\partial x} \right) + \frac{\partial}{\partial x} \left( \eta_1(x) \frac{\partial^2 u_1(x,t)}{\partial x \partial t} \right) \\
\rho_2(x) \frac{\partial^2 u_2(x,t)}{\partial t^2} &= \frac{\partial}{\partial x} \left( g_2(x) \frac{\partial u_2(x,t)}{\partial x} \right) + \frac{\partial}{\partial x} \left( \eta_2(x) \frac{\partial^2 u_2(x,t)}{\partial x \partial t} \right) \\
\rho_3(x) \frac{\partial^2 u_3(x,t)}{\partial t^2} &= \frac{\partial}{\partial x} \left( g_3(x) \frac{\partial u_3(x,t)}{\partial x} \right) + \frac{\partial}{\partial x} \left( \eta_3(x) \frac{\partial^2 u_3(x,t)}{\partial x \partial t} \right) \\
... \\
\rho_i(x) \frac{\partial^2 u_i(x,t)}{\partial t^2} &= \frac{\partial}{\partial x} \left( g_i(x) \frac{\partial u_i(x,t)}{\partial x} \right) + \frac{\partial}{\partial x} \left( \eta_i(x) \frac{\partial^2 u_i(x,t)}{\partial x \partial t} \right)
\end{align*}
\]

Continuity conditions:
\[ u_1(x_1,t) = u_2(0^+,t) \]
\[ \partial_+ u_1(x_1,t) = \partial_- u_2(0^+,t) \]
... 

Boundary conditions:
\[ u_1(0,t) = \text{input signal} \]
\[ u_2(x,t) = \text{output signal} \]

Initial conditions:
\[ u_1(x,0) = 0 \]
\[ \partial_+ u_1(x,0) = 0 \]

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ANY QUESTIONS?

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