CE191: Civil and Environmental Engineering Systems Analysis

Lab 1: Water Network Supply Optimization

Due: Wednesday 09/23/09 1:00pm

Professor A. Bayen *Fall 09*

1 Lab Overview

In this lab, you will learn to formulate a linear program to determine the optimal (cheapest) assignment of water from water sources to destination cities. Section 2 provides a description of the full problem and the constraints. In Section 3, you are asked to form the linear program, and implement it in matlab. Finally, in Section 4, you will modify the linear program to analyze various scenarios. **Please remember to submit your MATLAB code (.m files in one ZIP file), and explain in the report how to run the code.**

2 Problem Description

The county of Orchard is growing in population, and the public works department predicts the need for an increased water supply. There are five towns in the county, named Appletown, Berrytown, Cherrytown, Grapetown, and Mangotown.

Currently, the water is drawn from two reservoirs (Majorlake and Minorlake) on local rivers of good quality but insufficient quantity. Future sources, the ones to be tapped next, have various problems. Water from a nearby aquifer is available in large quantities, but its hardness is too high unless it is blended with a lower hardness source. The maximum allowed total mass of hardness in kilograms per million liters is 1,200.

Water from a distant stream is of adequate quality, but a pipeline has to be built, which results in a high cost to pump the water to the treatment plant.

The city is planning for the predicted demand ten years from the present. Table 1 lists the expected water needs of the five towns at that time (in million liters per day):

Town	Appletown	Berrytown	Cherrytown	Grapetown	Mangotown
Water Needs	30	10	50	20	40

Table 1: Expected water needs (in mld) in ten years.

The four sources are numbered as follows: 1- Majorlake, 2- Minorlake, 3- Aquifer and 4- the distant stream. The supply limits in millions of liters per day, and the hardness in kilograms per million liters are given in Table 2.

	Source 1	Source 2	Source 3	Source 4
Supply Limit (`mld`)	15	10	60	80
Hardness (`kg/ml`)	200	200	2,300	700

Table 2: Characteristics of water depending on the Source.

Table 3 provides the costs to obtain water in dollars per million liters, depending on the source and the town. For example, the cost of providing water from Source 3 to Berrytown is 915 `$/ml` (in bold in Table 3).

	Appletown	Berrytown	Cherrytown	Grapetown	Mangotown
Source 1	450	460	440	445	455
Source 2	495	500	505	510	490
Source 3	900	**915**	885	920	920
Source 4	1800	1815	1795	1785	1820

Table 3: Cost of providing water, depending on the source and the town ($/ml).

Figure 1 shows the layout of the pipes between the sources and the towns. Note that the pipes linking source 1 to the supply network and source 2 to the supply network merge downstream of source 1 and source 2.

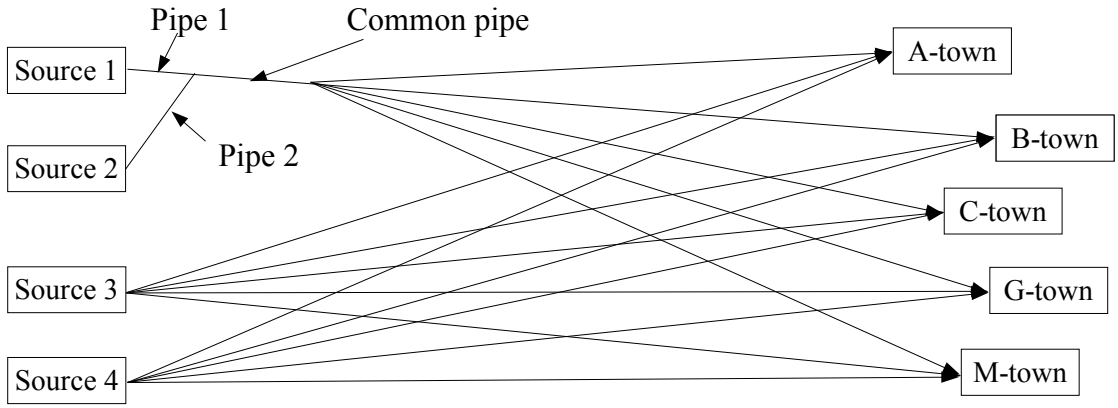

Figure 1: Layout of pipes between sources and towns (not to scale).

For structural reasons, if the flow in pipe 1 is greater than 10 mld, then the flow in the common pipe must not be greater than 20 mld.

3 Implementation

Question 3.1 Formulate a Linear Program (LP) that the county of Orchard can use to minimize the costs of obtaining its water needs in ten years, while satisfying the legal and structural constraints described above. Define your notation carefully and precisely.

Question 3.2 Solve the LP that you have formulated using MATLAB. Provide the value of the objective function and the value of the decision variables at optimum. Describe qualitatively the optimal solution. Which constraints are binding and which are not?

4 Additional Analysis

The purpose of the following questions is to study the influence of changes in the parameters compared to the original Linear Program solved in the previous question. Each question and subquestion is independent from the other ones, i.e. the changes are not cumulative.

Where applicable, provide the value of the objective function as well as the value of the decision variables.

Question 4.1 The actual demand and supply limits may differ from the forecasts mentioned earlier.

1. Solve the Linear Program with the demand of water for each town 7% lower than the forecast described earlier. Explain what happens (no more than three sentences).
2. Solve the Linear Program with the demand of water for each town 7% higher than the forecast described earlier. Explain what happens.
3. The supply limits for sources 3 and 4 are 95 and 75 mld respectively (instead of 60 and 80 mld). Solve the Linear Program and explain what happens.

Question 4.2 Kiwi Inc., a large factory, is outside the county of Orchard but close to Berrytown. It does not currently receive any water from the county of Orchard. The factory would like to buy water from the county of Orchard in the future (ten years). The factory accepts to be responsible for the investment to join the water supply network of the county of Orchard, and is ready to sign a contract with the county to buy 1 mld for the price of \$1,600 per day. The county of Orchard expects to incur the same costs to supply the water to the factory as it does to supply the water to Berrytown. Should the county of Orchard accept these conditions?[1]

Question 4.3 The county of Orchard has the option to increase the supply limit of Source 1 by adding a small supporting structure. The small structure needed to increase the limit by 2 mld would have to be replaced every year and costs approximately \$20,000. We can assume that this yearly cost is equivalent to a daily cost[2] of $20,000/365 \approx \$55$. Should the county choose this option? Note that this small structure does not change the flow limit in the common pipe downstream of Sources 1 and 2.

Question 4.4 The county of Orchard has the option to increase the supply limit of Source 3. What is the maximum daily cost the county should be willing to pay in order to increase the supply limit of Source 3 (in the same manner as in the previous question, we can assume that the cost for upgrading the facility can be viewed as a daily cost)?

[1] We assume that the County does not want to lose money when supplying water.
[2] To simplify, we are neglecting the time value of money.