Lecture 2: graphical solutions of LPs

- Graphical interpretation of constraints
- Feasible set
- Gradient of the cost function
- Unbounded feasible set
- Unbounded cost function
- Infeasibility

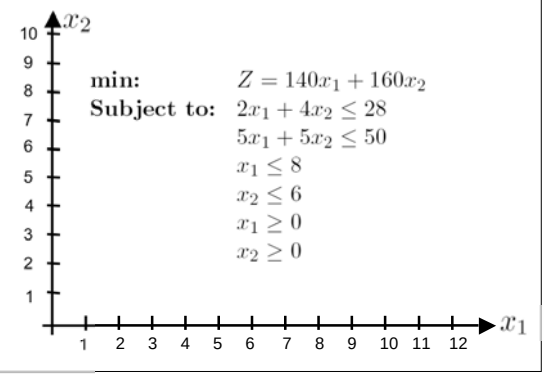
Graphical solutions of linear programs

Example from the textbook [Chap 3., p. 43]

$$
\begin{aligned}
\textbf{min:} \quad & Z = 140x_1 + 160x_2 \\
\textbf{Subject to:} \quad & 2x_1 + 4x_2 \leq 28 \\
& 5x_1 + 5x_2 \leq 50 \\
& x_1 \leq 8 \\
& x_2 \leq 6 \\
& x_1 \geq 0 \\
& x_2 \geq 0
\end{aligned}
$$

[Civil and Environmental Engineering, Revelle, Whitlatch and Wright]

Construction of the feasible set



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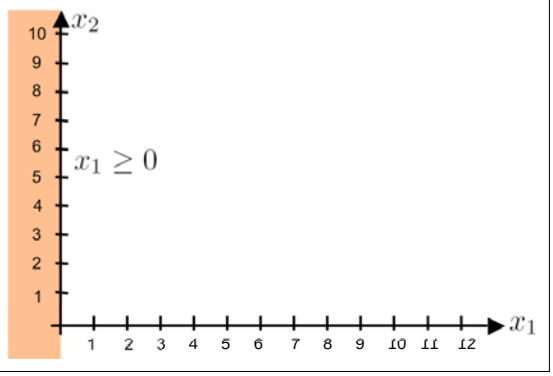

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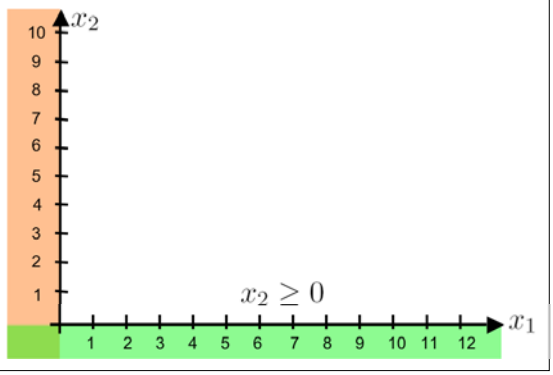

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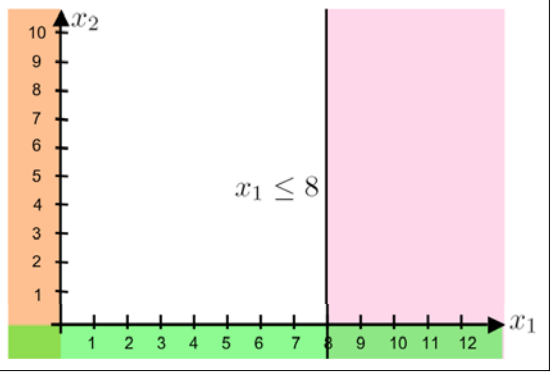
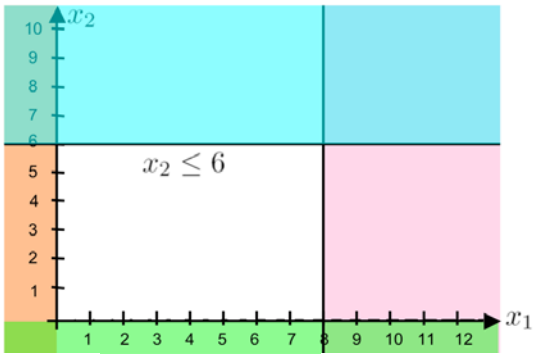
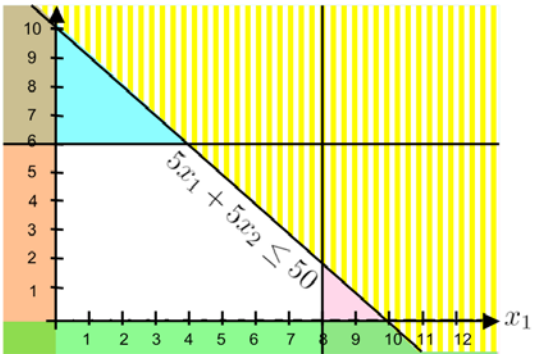


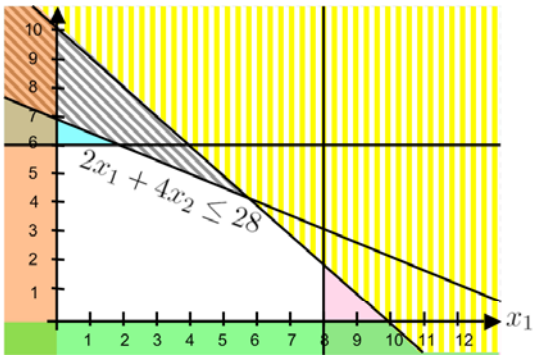
Construction of the feasible set



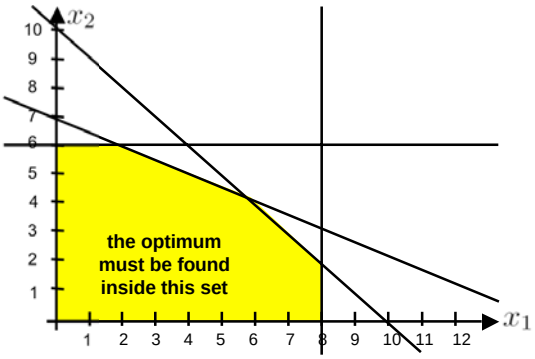
Construction of the feasible set



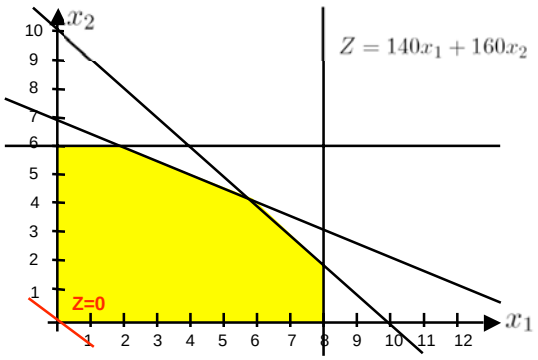
Construction of the feasible set



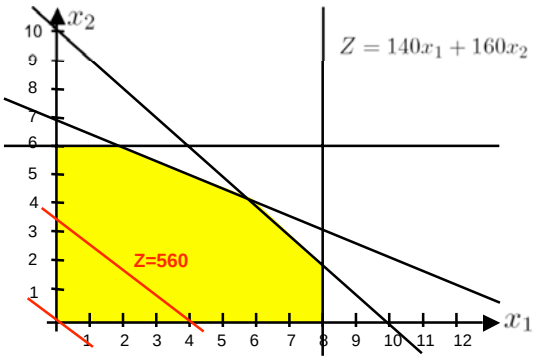
Feasible set: result



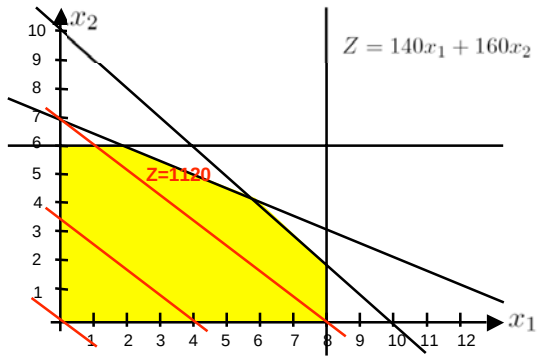
Optimal solution: isolines



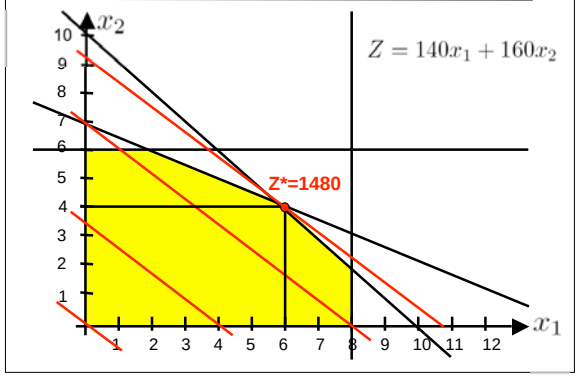
Optimal solution: isolines



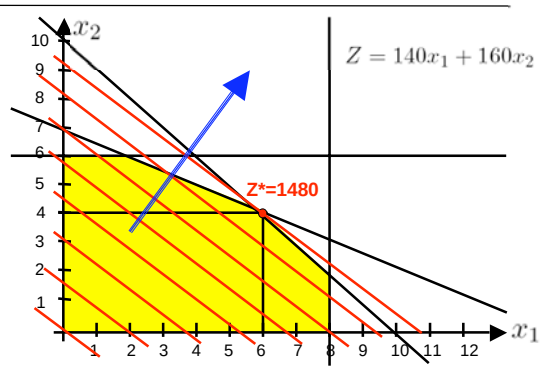
Optimal solution: isolines



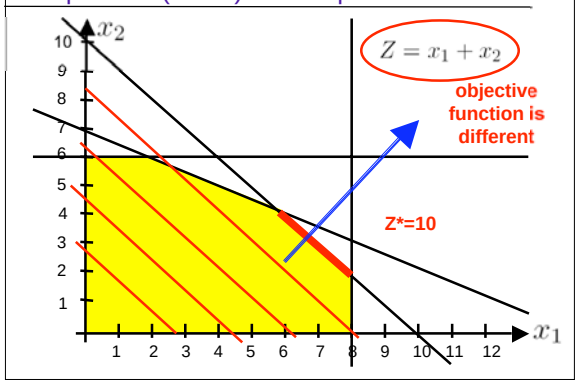
Optimal solution: isolines



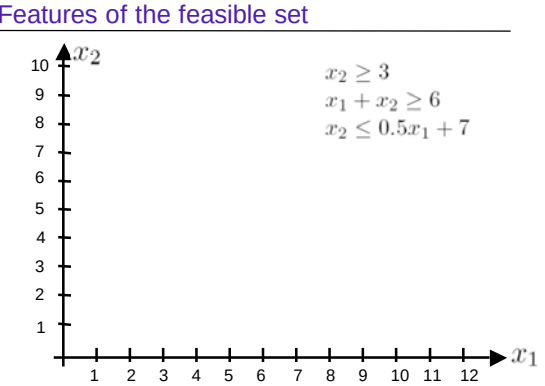
Gradient of the cost function



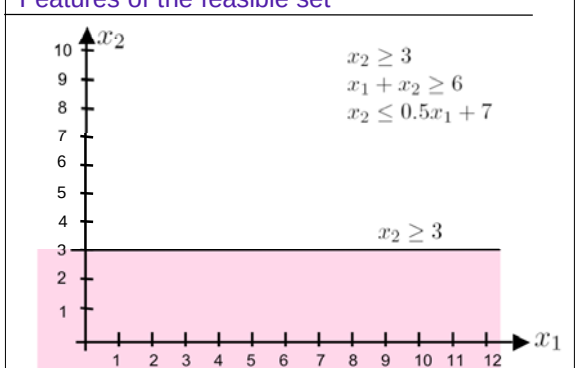
Uniqueness (or not) of the optimum



Features of the feasible set



Features of the feasible set



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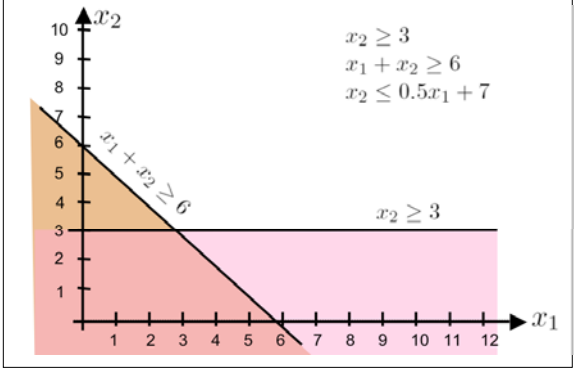

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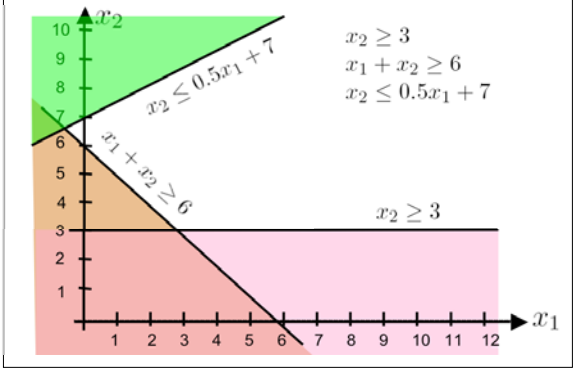
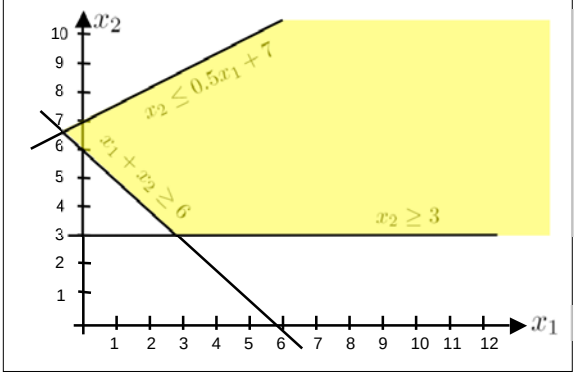
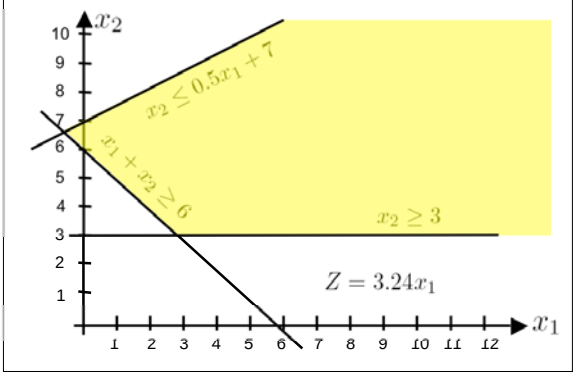


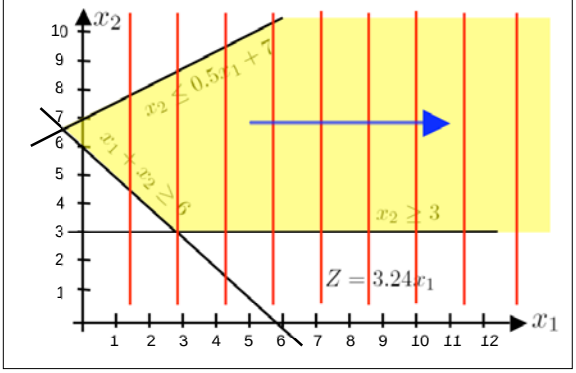
Feasible set is unbounded



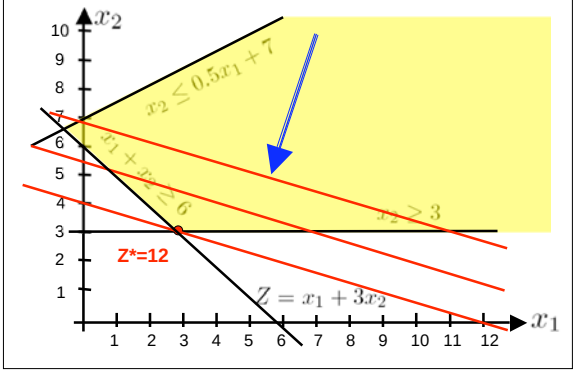
Objective function might be unbounded too



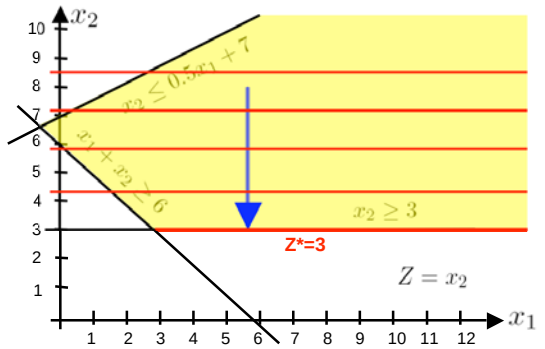
Objective function might be unbounded too



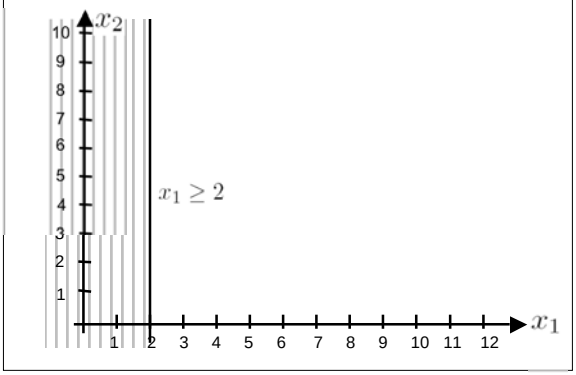
Objective might be bounded



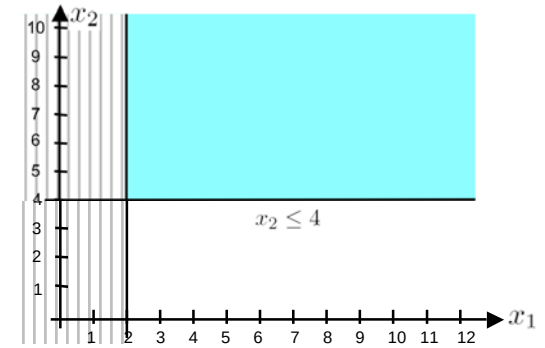
Objective might not be unique



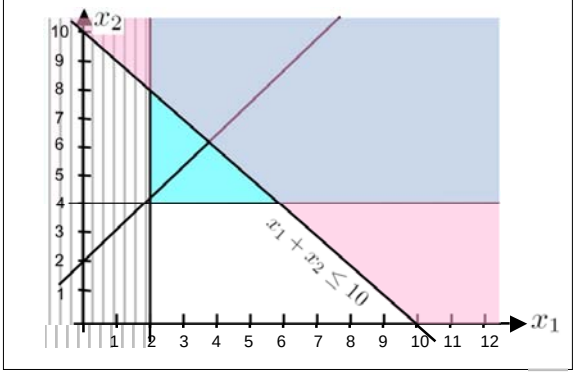
Feasible set might be empty



Feasible set might be empty



Feasible set might be empty



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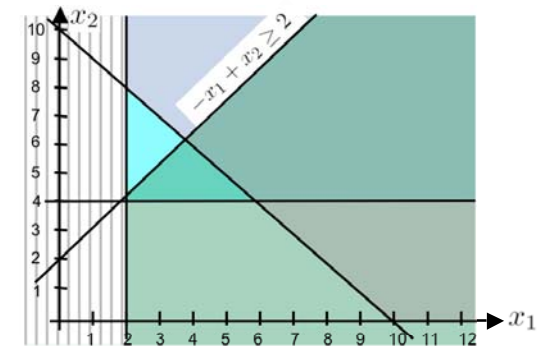

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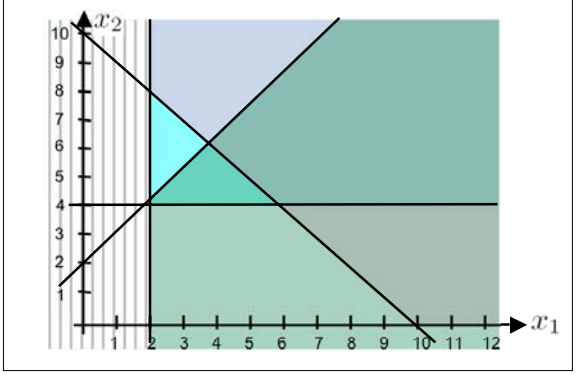

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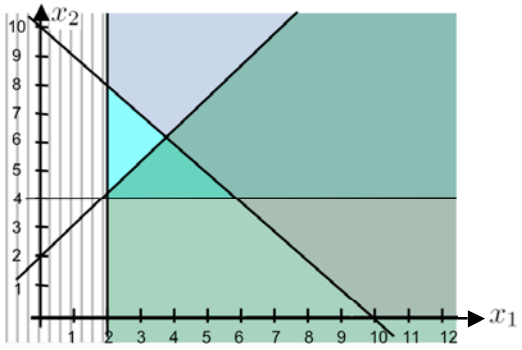
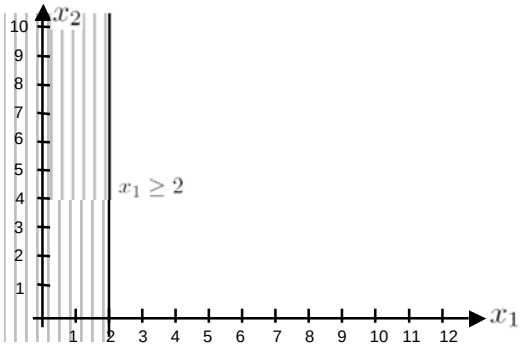
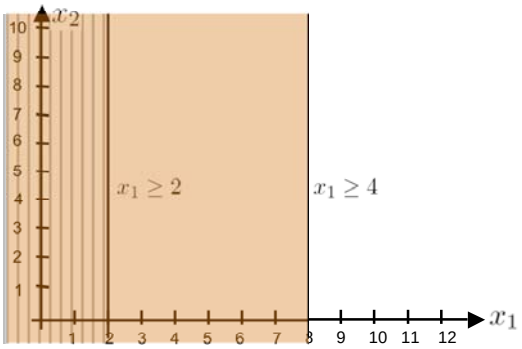


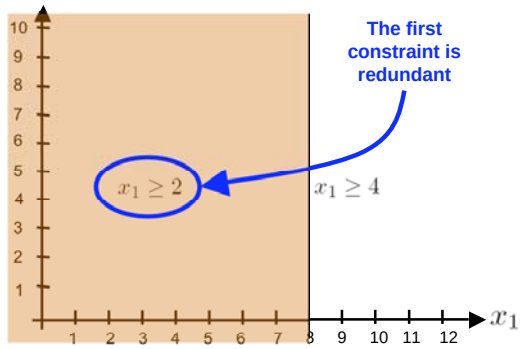
Redundant constraints



Redundant constraints



Redundant constraints



Graphical solution of LPs: general method

- Write your LP
- Successively eliminate half spaces corresponding to your constraints
- Is the feasible set empty?
  - YES → problem infeasible
  - NO → is the feasible set bounded?
    - NO → is solution finite?
      - NO: → finished
      - YES → is there a unique solution?
        - YES → corner point → finished
        - NO → face →finished
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